Uncertainty Quantification for a Microstrip Self-Biased Ku-band Circulator

Yoan Le Noane, Evan Roué, Norbert Parker, Mihai Telescu, Vincent Laur, Noël Tanguy

Uncertainty Quantification for a Microstrip Self-Biased Ku-band Circulator

Yoan Le Noane[#], Evan Roué[#1], Norbert Parker[#], Mihai Telescu[#2], Vincent Laur[#], Noël Tanguy[#]

[#] Univ Brest, Lab-STICC, CNRS, UMR 6285, F-29200 Brest, France

{[1]Evan.Roue, [2]Mihai.Telescu}@univ-brest.fr

***Abstract* — This paper focuses on uncertainty quantification for a Y-junction circulator. In the process of transitioning from prototyping to mass-production it is very important to understand how the uncertainties affecting input design parameters impact crucial features of the device behavior such as bandwidth or resonant frequency. This paper demonstrates the construction of a polynomial chaos expansion metamodel and its use in performing sensitivity analysis. The impact of the various sources of uncertainty affecting the circulator's behavior is quantified and the results are discussed.**

***Keywords* — circulator, polynomial chaos expansion, self-biased device, stochastic analysis**

I. Introduction

Circulators are commonly found in single antenna full-duplex systems or in RF front-ends to provide protection against impedance mismatches. The non-reciprocal behavior is obtained using a magnetically saturated ferrite whose permeability is thus anisotropic. Despite good electrical performance, these devices are known for their bulkiness. Indeed, a magnet is required to magnetize the soft ferrite. Furthermore, they are relatively expensive due to their complicated hybrid-manufacturing process (insertion of ferrite pucks in a stripline or microstrip structure).

One of the most promising approaches to mitigate these drawbacks is the use of a pre-oriented hexaferrite substrate [1]. This ferrite type is able to maintain a strong remanent magnetization M_r (without applying a static magnetic field) while also featuring a high anisotropy field H_k. These properties allow for the development of self-biased circulators. In doing so, a thickness reduction of up to more than 90% can be achieved with respect to commercial circulators using a soft ferrite magnetized by a permanent magnet. However, precise characterization of the electromagnetic properties of hexaferrites is challenging, in particular the evaluation of the relative permittivity ε_r and the anisotropy field H_k. Furthermore, sensitivity to the metallization dimensions has to be studied and quantified.

To better understand uncertainty propagation due to both material properties and device geometry, a stochastic approach was devised. Using data collected from a full-wave simulator, polynomial chaos metamodels were constructed and used to predict how the variability of design parameters impacts S-parameter response.

II. Circulator Industrialization: Problem Overview

The circulator topology used in this study and illustrated in Fig. 1 is a self-biased microstrip Y-junction resonator with quarter-wavelength impedance matching lines. The substrate is a hexaferrite sheet. Nominal values for material characteristics and design parameters cannot be disclosed. However, the tolerances are given in Table 1. Dimensions l_a, $R_{junction}$, w_{50}, w_a are affected by the tolerances of the metallization; variation in substrate height $h_{substrate}$ is also considered. The uncertainties related to the hexaferrite mainly impact the anisotropy field H_k and the relative permittivity ε_r.

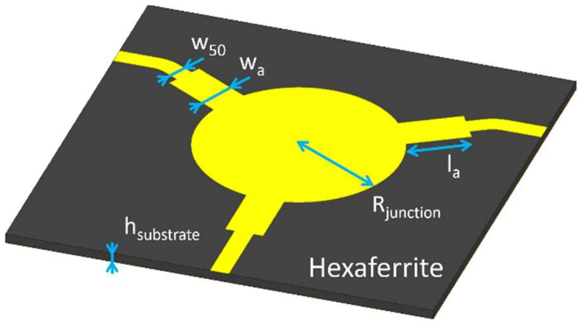

Fig. 1. Analyzed microstrip Y-junction circulator topology

Table 1. Uncertainties of design parameters and material properties.

Metallization	$h_{substrate}$	ε_r	$H_k\ (kOe)$
± 3 µm	± 10 µm	± 0.5	± 5.5 %

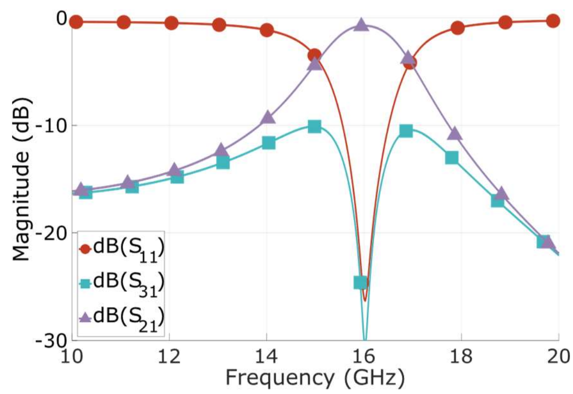

Fig. 2. Simulated S-parameters for the nominal circulator design (return loss in red/circles, isolation in cyan/squares, transmission in purple/triangles).

In the early stages of the design flow, the resonator radius ($R_{junction}$) and the quarter-wavelength line width (w_a) of the circulator were determined using Bosma's theoretical model [2] for a central frequency of 16 GHz. The width of the input lines (w_{50}) was computed in order to guarantee a characteristic impedance of 50 Ω. This topology was subsequently optimized using the Ansys HFSS™ electromagnetic solver. The obtained S-parameter response is given in Fig. 2. Both adaptation and isolation achieve good levels with peaks below $-20\ dB$ at the desired frequency of 16 GHz.

In order to qualitatively illustrate the effects of variability, 10 simulations, with the uncertain parameters varying randomly within the prescribed tolerances, were carried out and the S-parameter responses were plotted in Fig. 3. The quantitative analysis is discussed in the following section.

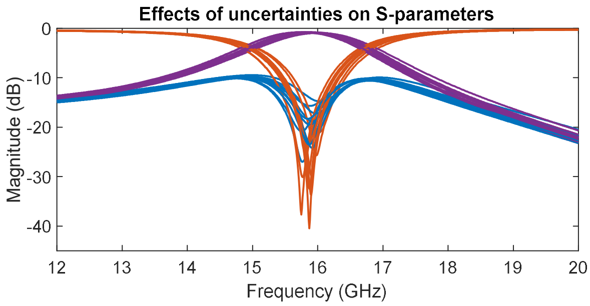

Fig. 3 The effects of variability on S-parameters (return loss in red, isolation in cyan, transmission in purple).

III. Stochastic Metamodeling

A. Polynomial chaos approach

Traditionally, uncertainty analysis was almost synonymous of Monte Carlo (MC) analysis: brute force stochastic investigation in which design parameters are varied randomly according to their respective tolerances. In order to properly observe quality metrics this process requires a very large number of simulations and entails a great computational cost, especially when it involves a full-wave electro-magnetic solver.

A solution allowing to circumvent the problem is the use of Polynomial Chaos Expansion (PCE). A popular tool implementing this approach is the UQLab toolbox [3].

PCE considers the random vector $\boldsymbol{X} \in R^N$ which includes variables such as design parameters or material characteristics, and attempts to predict the behavior of random variable Y which is a quality metric, assumed to have finite variance by means of a metamodel $\mathcal{M}$ defined as

$$Y = \mathcal{M}(\boldsymbol{X}) = \sum_{\alpha \in \mathbb{N}^M} z_\alpha \Psi_\alpha(\boldsymbol{X}) \qquad (1)$$

where z_α are the coefficients associated to the multivariate, orthonormal polynomials Ψ_α which form a basis of the Hilbert space. Index vector $\boldsymbol{\alpha}$ contains ordered lists of integers such that $\boldsymbol{\alpha} = (\alpha_1, \cdots, \alpha_M)$ and is associated with Ψ_α through

$$\Psi_\alpha(\boldsymbol{X}) = \prod_{i=1}^{M} \psi_{\alpha_i}(X_i) \qquad (2)$$

where ψ_{α_i} are the univariate polynomials chosen according to the distribution of X_i. If the input random variables follow uniform distributions, Legendre polynomials are usually chosen.

In practice, the space α is reduced to a set $\mathcal{A}$ by limiting the total degree of a multivariate polynomial $\alpha_{tot} = \sum_{i=1}^{M} \alpha_i$ to a degree p (usually 3 to 5) according to one of the truncation schemes described in [4].

The construction of the metamodel requires a training phase. Variable Y is computed for a reduced set of random values of $\boldsymbol{X}$. With this knowledge available, the computation of coefficients z_α follows. Depending on the application, this operation may prove non-trivial and several approaches are possible. The example presented in this paper required the use of Bayesian compressive sensing [5].

B. Validation and numerical results

In the specific case of the device under study, vector $\boldsymbol{X}$ contains the input variables listed in Table 1 and illustrated in Fig. 1. A total of 7 variables is thus considered: l_a, $R_{junction}$, w_{50}, w_a, $h_{substrate}$, H_k and ε_r.

Four different output metrics Y were studied: the best isolation frequency, the best adaptation frequency, the bandwidth for isolation (absolute value measured at -12 dB) and the bandwidth for adaptation (also absolute value at -12 dB). These quantities are subsequently noted f_I, f_{RL}, B_I and B_{RL}, respectively.

In order to study uncertainty propagation, the authors constructed PCE metamodels from data obtained via HFSS™ simulation using 150 identification runs. Another 150 runs were then used for validation: probability density functions were plotted using the validation data directly and the prediction derived from the metamodels, they are shown in Fig. 4 and Fig. 5.

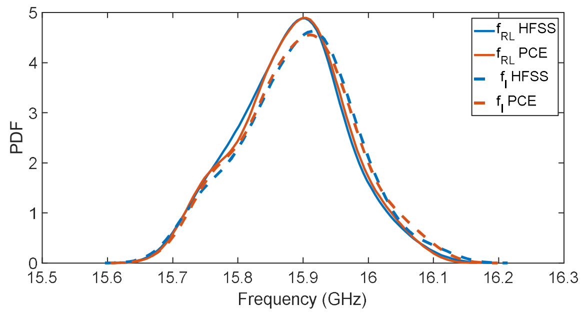

Fig. 4 Statistical analysis on f_{RL} (solid line) and f_I, (dashed line): HFSS™ (cyan) vs. PCE (red).

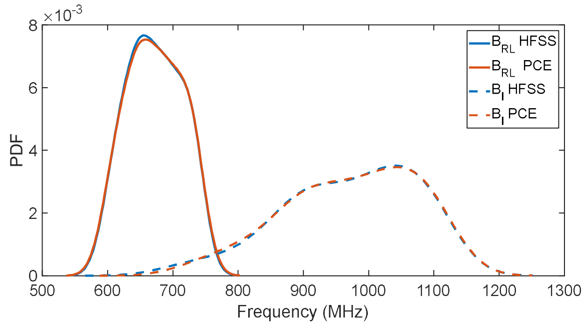

Fig. 5 Statistical analysis on the bandwidth at -12 dB: HFSS™ (cyan) vs. PCE (red). Solid line for B_{RL}, dashed line for B_I.

There is excellent agreement between the prediction and reference. The value of the metamodel comes from its computational advantage. Indeed 150 runs take approximately 2 hour 35 minutes using HFSS with 6 parallel simulations. The evaluation of the PCE models in 150 points only takes 0.9 seconds. The hardware used is an AMD Ryzen 9 3900X, 64 GB of ram and a NVMe solid state drive.

It is important to understand the parametric nature of the polynomial chaos metamodel. It can be evaluated at great speed for any combination of the parameters illustrated in Fig. 1. Consequently, should the tolerances in Table 1 be reduced as industrialization progresses there is no use to build a new metamodel: a new statistical assessment can be carried-out simply by using the existing metamodel with the new tolerance values. Moreover, the availability of the metamodel makes accurate variance analysis possible, the benefits of this investigation are shown in the following section.

IV. Sensitivity analysis

The focus of this section is understanding and quantifying the impact of various input variables on an output metric. Sensitivity analysis usually comes in the form of variance analysis and entails the computation of the so-called Sobol' indices [6].

Under a few reasonable assumptions, output variable Y can be expressed as

$$\begin{aligned} Y &= f(X_1, \dots, X_N) \\ &= f_0 + \sum_{j=1}^{N} f_j(X_j) \\ &\quad + \sum_{1\le j<k\le N} f_{jk}(X_j, X_k) + \cdots + f_{1,2,\dots,N}(X_1, \dots, X_N) \end{aligned} \tag{3}$$

where the first term f_0 is a constant that is equal to the expected value of $f(\boldsymbol{X})$ and, furthermore

$$Var(Y) = \sum_{j=1}^{N} D_j + \sum_{1\le j<k\le N} D_{j,k} + \cdots + D_{1,2,\dots,N} \tag{4}$$

where the first summation essentially designates the variance due to the effects of each unique variable X_j. The second summation represents the residual influence due conjointly to X_j and X_k but not including D_j and D_k. Further orders can be considered but in practice they are negligible most of the time.

First order Sobol' indices are defined as

$$S_j = \frac{D_j}{Var(Y)} \tag{5}$$

Depending on the application, accurate computation of the indices may require tens of thousands of simulations. A detailed discussion of this topic is beyond the scope of the present paper, but it is important to note that in our case 10^5 simulations were needed. This operation required little computational effort with the metamodels, but could not have been carried-out in practice with a full-wave simulator.

Fig. 6 and Fig. 7 provide an important takeaway for circulator designers and manufacturers. Uncertainties affecting hexaferrite material properties such as permittivity or anisotropy field have far greater impact than those affecting the circuit geometry. It appears obvious that improving the accuracy of material characterization and closely controlling the production process of the hexaferrite has far greater importance than reducing the geometric tolerances of the metallization. This was expected and is inline with the theory behind circulator design, but the interest of the present paper is that it quantifies uncertainty effects. Furthermore, note that when investigating bandwidth, the results show that the anisotropy field is the single dominant source of variability.

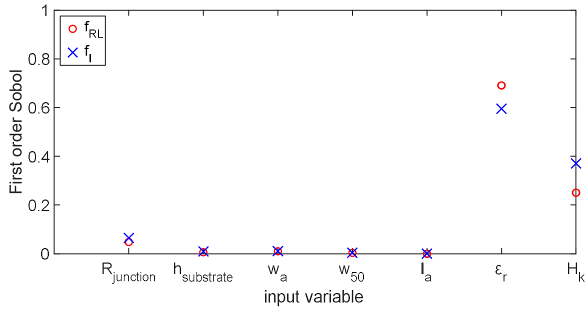

Fig. 6 First order Sobol' indices for on f_{RL} (red circles) and f_I, (blue crosses).

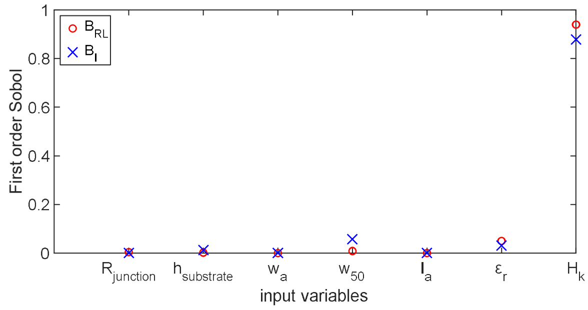

Fig. 7 First order Sobol' indices for B_{RL} (red circles) and B_I (blue crosses).

V. Conclusion

In this paper the authors studied uncertainty propagation in a Y-junction circulator in preparation of the device's industrialization. The practicality of the polynomial chaos approach using Bayesian compressive sensing was demonstrated. The resulting metamodels were used to produce comprehensive statistical assessments at negligible computational cost. Variance analysis was conducted and the impact of the different sources of uncertainty was quantified.

References

[1] N. Parker-Soues *et al.*, "Compact Self-Biased Q-Band Circulators," *IEEE Trans. Magn.*, vol. 57, no. 4, pp. 1–6, Apr. 2021, doi: 10.1109/TMAG.2021.3059946.

[2] H. Bosma, "On Stripline Y-Circulation at UHF," *IEEE Trans. Microw. Theory Tech.*, vol. 12, no. 1, Art. no. 1, Jan. 1964, doi: 10.1109/TMTT.1964.1125753.

[3] S. Marelli and B. Sudret, "UQLab: A Framework for Uncertainty Quantification in Matlab," pp. 2554–2563, Jul. 2014, doi: 10.1061/9780784413609.257.

[4] B. Sudret, "Polynomial chaos expansions and stochastic finite element methods," in *Risk and Reliability in Geotechnical Engineering*, Boca Raton, FL, USA: CRC Press, 2014, pp. 265–300.

[5] S. Ji, Y. Xue, and L. Carin, "Bayesian Compressive Sensing," *IEEE Trans. Signal Process.*, vol. 56, no. 6, pp. 2346–2356, Jun. 2008, doi: 10.1109/TSP.2007.914345.

[6] I. M. Sobol', "Sensitivity analysis for non-linear mathematical models," *Math Model. Comput Exp*, vol. 1, pp. 407–414, 1993.