Detection of Spread Spectrum Transmissions using fluctuations of correlation estimators

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DETECTION OF SPREAD SPECTRUM TRANSMISSIONS USING FLUCTUATIONS OF CORRELATION ESTIMATORS

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ABSTRACT

A method for detection of a Direct-Sequence Spread Spectrum (DS-SS) communication hidden in the noise is proposed. Indeed, DS-SS signals are well-known for their low probability of interception: their statistics are similar to the statistics of a noise, and, furthermore, they are usually transmitted below the noise level. The proposed method is based on the fluctuations of autocorrelation estimators. We show that, when a DS-SS signal is hidden in the noise, these fluctuations increase. A theoretical analysis shows that detection is possible, even with very low signal to noise ratio at the detector input. Experimental results are provided to illustrate the approach. The method is also able to estimate the symbol period of the DS-SS signal.

1. INTRODUCTION

Spread spectrum transmissions use a bandwidth considerably larger than the minimal required bandwidth [4]. Their interests are:

- In the military domain, they allow transmission with a very low power spectral density (PSD). Hence, they are difficult to detect.
- In the non-military domain, they allow many transmitters to share the same frequency band with low interference (e.g. Code Division Multiple Access [2]). They are also robust with respect to echoes.

While used in the military domain for decades [1][7], spread spectrum is now taking a growing importance in non-military transmissions [3]. The result is a proliferation of low-cost spread spectrum transmission devices. The low probability of interception of spread spectrum signals is a problem for authorities in charge of spectrum surveillance. In this paper, we propose a method to detect a spread spectrum transmission far below the noise level. The paper is organized as follows: in Section 2, we briefly recall the principle of Direct-Sequence Spread Spectrum transmission (DS-SS). Our approach for blind detection of a DS-SS signal is described in Section 3. Then, a theoretical analysis is developped in Section 4: it proves that the method works even when the signal to noise ratio is very low. Then, experimental results are shown to illustrate the approach (Section 5).

2. PRINCIPLE OF DIRECT SEQUENCE SPREAD SPECTRUM TRANSMISSIONS

In a DS-SS transmission, the symbols a_k are multiplied by a pseudo-random sequence which spreads the bandwidth [6]. The pseudo-random sequence, as well as the carrier and symbol frequencies, are known by the receiver. The receiver correlates the received signal with the pseudo-random sequence, in order to retrieve the symbols [5]. A receiver which does not know these parameters cannot even detect the presence of a DS-SS signal, because it is usually under the noise level. The signal at the output of the receiver filter is:

$$y(t) = s(t) + b(t) \tag{1}$$

where $b(t)$ is the noise at the output of the receiver filter $g(t)$, and $s(t)$ is the filtered noise-free DS-SS signal.

$$s(t) = (g * \widetilde{s})(t) \tag{2}$$

$$b(t) = (g * \widetilde{b})(t) \tag{3}$$

where $\widetilde{b}(t)$ is the received noise and $\widetilde{s}(t)$ the noise-free received DS-SS signal.

$$\widetilde{s}(t) = \sum_{k=-\infty}^{+\infty} a_k h(t - kT_s) \tag{4}$$

$$h(t) = \sum_{k=0}^{P-1} c_k p(t - kT_c) \tag{5}$$

$p(t)$ is the convolution of the transmission filter and the channel filter (which represents the channel echoes).

$\{c_k, k = 0, ..., P-1\}$ is the pseudo-random sequence, T_c is the chip period, and T_s is a symbol period ($T_s = P.T_c$).

In the sequel, γ will stand for power spectral density, and the following hypotheses will be assumed:

- The symbols are centered and uncorrelated.
- The received noise $\widetilde{b}$ is white, gaussian, centered, and uncorrelated with the signal. Its power spectral density is $\frac{N_0}{2}$
- The time extension of $(g * h)(t)$ is only a little more than T_s (this is always the case, otherwise it would mean that the receiver bandwidth is extremely small with respect to the signal bandwidth).
- The signal to noise ratio (in dB) at the output of the receiver filter is negative (the signal is hidden in the noise).

3. PROPOSED APPROACH

We recall that we consider a non-cooperative context, hence no a priori information is available (the spreading sequence, the symbol period, etc., are unknown).

The basic principle of any detection method (whatever the application is) is to take profit of the fact that the signal statistical properties are not the same as the noise statistical properties. For instance, in some simple applications, the signal and noise frequencies are not the same, hence filters are sufficient to detect the presence of a signal. Here, the application is much more complex, because a spread spectrum signal is specially built to be similar to a noise, in order to have a low probability of interception (remind that spread spectrum was initially developed for military applications). For instance, the autocorrelation of a spread spectrum signal is close to a Dirac function, as well as the autocorrelation of a white noise (this is due to the pseudo-random sequence).

The originality of the proposed approach is to be based on the fluctuations of autocorrelation estimators, instead than on the autocorrelation itself. Although the autocorrelation of a DS-SS signal is similar to the autocorrelation of a noise, we will prove that the fluctuations of estimators are totally different.

In order to compute the fluctuations, we must divide the received signal into temporal windows. We will note T the window duration and M the number of windows. An autocorrelation estimator is applied to each window, then the fluctuations are computed.

Using window number n, we compute an estimation of the correlation:

$$\widehat{R^n_{yy}}(\tau) = \frac{1}{T}\int_0^T y(t)y^*(t-\tau)dt \tag{6}$$

Using M windows, we can estimate the second order moment of the estimated correlation $\widehat{R_{yy}}(\tau)$:

$$\rho(\tau) = \widehat{E}\{|\widehat{R_{yy}}(\tau)|^2\} = \frac{1}{M}\sum_{n=0}^{M-1} |\widehat{R^n_{yy}}(\tau)|^2 \tag{7}$$

$\rho(\tau)$ is a measure of the fluctuations of $\widehat{R_{yy}}(\tau)$. In the paragraphs below, we show that this measure is a powerful tool to detect the presence of a spread spectrum signal hidden in the noise.

Since the noise and the signal are uncorrelated, we can write:

$$\widehat{R_{yy}}(\tau) \simeq \widehat{R_{ss}}(\tau) + \widehat{R_{bb}}(\tau) \tag{8}$$

The next Section explains why the method works, and show that it can work even when the SNR is very low.

4. THEORETICAL ANALYSIS

We will successively investigate the contribution of the noise and of the signal. But, before, we need a theoretical result about the second order moment of a correlation estimator.

For readers who are not familiar with statistics, probabilities, and signal processing, it is recommended to have a look to the experimental results (Section 5) first, in order to have a better understanding of the meaning of some measures.

4.1. Second order moment of a correlation estimator

The estimator of the correlation between $u(t)$ and $v(t)$ is:

$$\widehat{R_{uv}}(\tau) = \frac{1}{T}\int_0^T u(t)v^*(t-\tau)dt \tag{9}$$

Let us note:

- $d(t) = u(t)/T$ for $0 \le t \le T$ and $d(t) = 0$ elsewhere
- $e(t) = v^*(-t)$

We can write:

$$\widehat{R_{uv}}(\tau) = \int_{-\infty}^{+\infty} d(t)e(\tau - t)dt \tag{10}$$

This can be seen as the filtering of a signal $e(t)$ by a filter $d(t)$. Hence:

$$\gamma_R(\nu) = |D(\nu)|^2.\gamma_e(\nu) \tag{11}$$

where $D(\nu)$ is the Fourier transform of $d(t)$. If T is large enough, we obtain:

$$\gamma_R(\nu) = \frac{1}{T}.\gamma_u(\nu).\gamma_v(\nu) \tag{12}$$

Since $E\{|\widehat{R_{uv}}(\tau)|^2\}$ is the average power of $\widehat{R_{uv}}(\tau)$ (i.e. the integral of its PSD), we can write:

$$E\{|\widehat{R_{uv}}(\tau)|^2\} = \frac{1}{T}\int_{-\infty}^{+\infty} \gamma_u(\nu).\gamma_v(\nu)d\nu \tag{13}$$

4.2. Contribution of the noise

In this subsection, we will consider noise alone (no spread spectrum signal is hidden in the noise). Since the noise is random, the fluctuations of the autocorrelation estimator are random themselves. We will characterize them by their mean and their standard deviation.

4.2.1. Average value of the fluctuations $\rho_b(\tau)$

The average value of $\rho_b(\tau) = \widehat{E}\{|\widehat{R_{bb}}(\tau)|^2\}$ is the average of $|\widehat{R_{bb}}(\tau)|^2$. Let us note $m_\rho^{(b)}$ this average value. Using equation 13 we can write:

$$m_\rho^{(b)} = E\{|\widehat{R_{bb}}(\tau)|^2\} = \frac{1}{T}\int_{-\infty}^{+\infty} |\gamma_b(\nu)|^2 d\nu \tag{14}$$

Let us note $G(\nu)$ the Fourier transform of the receiver filter. We have:

$$\gamma_b(\nu) = |G(\nu)|^2\gamma_{\tilde{b}}(\nu) = \frac{N_0}{2}|G(\nu)|^2 \tag{15}$$

If the frequency response of the receiver filter is flat in $[-W/2, +W/2]$ and zero outside, it is easy to show that:

$$m_\rho^{(b)} = \frac{1}{T.W}\sigma_b^4 \tag{16}$$

where σ_b^4 is the noise variance.

4.2.2. Standard deviation of the fluctuations $\rho_b(\tau)$

The standard deviation of the fluctuations is:

$$\sigma_\rho^{(b)} = \sqrt{var\{\widehat{E}\{|\widehat{R_{bb}}(\tau)|^2\}} \tag{17}$$

Since the windows are independent, we have:

$$var\left\{\widehat{E}\{|\widehat{R_{bb}}(\tau)|^2\}\right\} = \frac{1}{M^2}\sum_{n=0}^{M-1} var\{|\widehat{R_{bb}^n}(\tau)|^2\} \tag{18}$$

Hence:

$$var\left\{\widehat{E}\{|\widehat{R_{bb}}(\tau)|^2\}\right\} = \frac{1}{M} var\{|\widehat{R_{bb}}(\tau)|^2\} \tag{19}$$

and:

$$var\{|\widehat{R_{bb}}(\tau)|^2\} = E\{|\widehat{R_{bb}}(\tau)|^4\} - \left(m_\rho^{(b)}\right)^2 \tag{20}$$

Due to its definition, the statistical behavior of $\widehat{R_{bb}}(\tau)$ is close to a gaussian because it is the average of a large number of random variables. Furthermore, except for small values of τ, its average value is null (the receiver filter creates a short term coherence in the noise, which can result in non-zero autocorrelation for τ small). Hence:

$$E\{|\widehat{R_{bb}}(\tau)|^4\} \simeq 3\left(m_\rho^{(b)}\right)^2 \tag{21}$$

So:

$$var\{|\widehat{R_{bb}}(\tau)|^2\} \simeq 2\left(m_\rho^{(b)}\right)^2 \tag{22}$$

Hence:

$$\sigma_\rho^{(b)} \simeq \sqrt{\frac{2}{M}}.m_\rho^{(b)} \tag{23}$$

4.3. Contribution of the signal

If we consider the spread spectrum signal alone, we show that high fluctuations of the autocorrelation estimator are obtained for each τ multiple of the symbol period. For simplicity, we will limit the proof to $\tau = T_s$. Generalization to $\tau = k.T_s$ is obvious.

We can write:

$$\widehat{R_{ss}}(T_s) = \frac{1}{T}\int_0^T s(t)s^*(t-T_s)dt \tag{24}$$

Let us note:

$$r(t) = (g * h)(t) \tag{25}$$

Using equations 2 and 4, and replacing, we obtain:

$$\widehat{R_{ss}}(T_s) = \frac{1}{T}\sum_{k=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} a_k a_m^* \int_0^T r(t-kT_s)r^*(t-(m+1)T_s)dt \tag{26}$$

Due to the limited time extension of $r(t)$, this expression simplifies to:

$$\widehat{R_{ss}}(T_s) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} a_k a_{k-1}^* \int_0^T |r(t-kT_s)|^2 dt \tag{27}$$

Let us note:

$$\sigma_a^2 = E\left\{|a_k|^2\right\} \tag{28}$$

and

$$\sigma_r^2 = \frac{1}{T_s}\int_0^{T_s} |r(t)|^2\,dt \tag{29}$$

Since the symbols are centered and independent, we have:

$$E\{|\widehat{R_{ss}}(T_s)|^2\} = \frac{1}{T^2}\sigma_a^4 \sum_{k=-\infty}^{+\infty}\left(\int_0^T |r(t-kT_s)|^2\,dt\right)^2 \tag{30}$$

That is:

$$E\{|\widehat{R_{ss}}(T_s)|^2\} = \frac{1}{T^2}\sigma_a^4\frac{T}{T_s}\left(T_s\sigma_r^2\right)^2 \tag{31}$$

and, since the signal power is:

$$\sigma_s^2 = \sigma_a^2\sigma_r^2 \tag{32}$$

we have:

$$m_\rho^{(s)} = E\{|\widehat{R_{ss}}(T_s)|^2\} = \frac{T_s}{T}\sigma_s^4 \tag{33}$$

We can note that:

$$\sigma_s^2 = \int_{-\infty}^{+\infty}\gamma_s(\nu)d\nu \tag{34}$$

and that:

$$\gamma_s(\nu) = |G(\nu)|^2\gamma_{\tilde{s}}(\nu) \tag{35}$$

4.4. SNR at detector output

In this section, we prove that the detector works even at very low SNR. The signal to noise ratio (in dB) at the detector output is:

$$SNR_{out} = 20\log\left(\frac{m_\rho^{(s)}}{\sigma_\rho^{(b)}}\right) \tag{36}$$

This is the ratio between the mean value of the peaks created by the DS-SS signal (if there is one such signal hidden in the noise), and the standard deviation of the estimator fluctuations due to the noise. One could object that it would be better to compare $m_\rho^{(s)}$ with $m_\rho^{(b)}$. In fact, this is not true, because it is really $\sigma_\rho^{(b)}$ which is significant to determine if the peaks due to signal may be hidden by the fluctuations due to noise. For instance, a large value of $m_\rho^{(b)}$ is not a problem if $\sigma_\rho^{(b)}$ is small. On the contrary, if $\sigma_\rho^{(b)}$ is large, the peaks due to signal can be totally hidden, even if $m_\rho^{(b)}$ is small.

For a receiver filter with flat frequency response in $[-W/2, +W/2]$ and zero outside, we have:

$$\frac{m_\rho^{(s)}}{\sigma_\rho^{(b)}} = W.T_s\sqrt{\frac{M}{2}}\left(\frac{\sigma_s^2}{\sigma_b^2}\right)^2 \tag{37}$$

Hence, the equation below gives the signal to noise ratio at the detector output as a function of input SNR and other parameters:

$$\begin{aligned} SNR_{out} \;&=\; 4.SNR_{in} + 20\log\left(W.T_s\right) \\ &\quad +10\log(M) - 10\log(2) \end{aligned} \tag{38}$$

The table below shows the results obtained with $W.T_s = 127$ and $M = 100$

$SNR_{in}(dB)$	$SNR_{out}(dB)$
-5	$+39$
-8	$+27$
-11	$+15$
-14	$+3$

(39)

Please note that increasing W does not always increase the output SNR. The optimum case is when W is equal to the DS-SS signal bandwidth (in that case WT_s is equal to the length of the pseudo-random sequence). Further increasing decreases SNR_{in} (and, consequently, it can decrease SNR_{out}), because more noise goes through the receiver filter. Since the characteristics of the DS-SS signal are not known a priori, the receiver filter bandwidth will usually be different to the optimum value. However, the tolerance is large: for instance, if W is twice the optimum value (i.e. an error of 100%), 3dB are lost in SNR_{in} and 6dB are gained in $20\log\left(W.T_s\right)$, hence 6dB are lost in SNR_{out}.

Equation 38 also shows that, from a theoretical point of view, the detector performances can be increased without limits, just by increasing the number of windows (M). However, on a practical point of view, we must take into account that the computation time is approximately proportional to M, hence the value of M cannot be increased without limits: it depends on the available computation power, and it depends also on the time allocated for detection.

5. EXPERIMENTAL RESULTS

Figure 1 shows an example of detector output. The curve represents $\rho(\tau)$ (i.e. the fluctuations of the autocorrelation estimator) as a function of τ (in μs). We can clearly see two peaks. This means that a DS-SS signal is hidden in the noise.

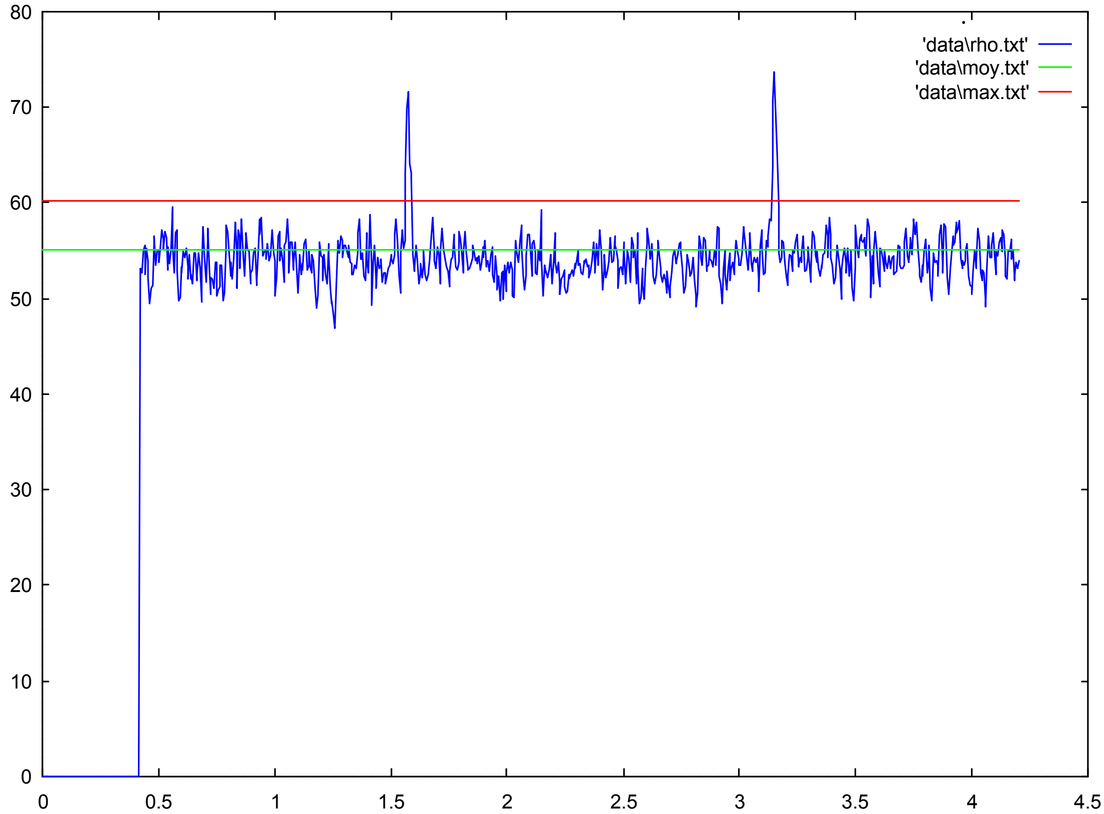

Figure 1: Example of detector output

Here, there was indeed a DS-SS signal hidden in the noise, and the detector input SNR was -10 dB (this is the SNR at the output of the receiver filter).

From the theoretical analysis, we know that the peaks are obtained for values of τ which are the multiples of the symbol frequency T_s. Hence, we can also determine the symbol frequency: $T_s = 1.54\mu s$.

The computation time to obtain this curve was 3s on a Intel Pentium II processor (266MHz), with a non-optimized C-program. The number of windows was $M = 200$, and we had $WT_s = 63$.

The horizontal lines show the theoretical average fluctuations ($m_\rho^{(b)}$, see eq. 16) and the theoretical average fluctuations plus 4 times the theoretical standard deviation on the fluctuations ($m_\rho^{(b)} + 4\sigma_\rho^{(b)}$. For $\sigma_\rho^{(b)}$ see eq. 23), if no DS-SS signal were hidden in the noise. Since the peaks are far above $m_\rho^{(b)} + 4\sigma_\rho^{(b)}$, there is no doubt that a DS-SS signal is hidden in the noise.

6. CONCLUSIONS

A method for detection of a Direct-Sequence Spread Spectrum communication hidden in a noise has been proposed. DS-SS signals are well-known for their low probability of interception: they are similar to noise and they are often transmitted below the noise level. We have shown that, although the autocorrelation of a DS-SS signal is the same as the autocorrelation of a noise, the fluctuations of correlation estimators are higher when a DS-SS signal is hidden in the noise. The method computes these fluctuations, and the fluctuation curve is displayed. This curve shows high equispaced peaks when a DS-SS signal is present.

This method is interesting in any non-cooperative context such as spectrum surveillance. Furthermore, the method is able to estimate the symbol period of the DS-SS signal. This information is required for Blind spreading sequence estimators, such as the method proposed in [8].

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