A fast ML-based receiver for MIMO Rician fading channel

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A FAST ML-BASED RECEIVER FOR MIMO RICIAN FADING CHANNEL

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Abstract:
We derive a fast maximum likelihood based (MLB) decoder for a multi-input multi-output (MIMO) Rician fading channel with additive white Gaussian noise (AWGN). The basic idea is to take profit of the Rician channel structure to significantly reduce the search of the optimum vector of symbols by the ML criterion. When channel diversity is low, we obtain bit error rates (BER) which are very close to the BER of the maximum likelihood (ML) optimum decoder. Comparisons in terms of BER for Quadrature Amplitude Modulation (QAM) are performed for the MLB, ML and OSIC (Ordered Successive Interference Cancellation) decoders via simulations. Finally, the ratio of computational complexities (depending of the number of transmitters and on the constellation), between the ML and the MLB is presented to show the interest of the proposed approach.

I INTRODUCTION

Narrowband MIMO transmission systems receive increased attention for a few years, due to their ability to provide large spectral efficiencies over rich scattering transmission channels. Spatial multiplexing systems, known as V-BLAST (Bell Laboratories Layered Space-Time) architecture, have been proposed recently, and first laboratory experiments have shown that spectral efficiencies as high as 20bits/s/Hz can be obtained [3].

The basic model of a narrowband MIMO transmission system is:

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{a} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{H}$ is the channel matrix, and $\boldsymbol{r}$, $\boldsymbol{a}$, and $\boldsymbol{n}$ respectively stand for the n_R-dimensional received vector, the n_T-dimensional transmitted vector (the entries of which are the symbols), and the noise vector. The objective of the receiver algorithm is to estimate $\boldsymbol{a}$ when $\boldsymbol{r}$ and $\boldsymbol{H}$ are known (in practice, $\boldsymbol{H}$ is estimated using a training sequence). We also assume[1] $E[\boldsymbol{a}\boldsymbol{a}^{\mathsf{H}}] = (p_0/n_T)\,\boldsymbol{I}_{n_T}$ with p_0 the total transmitted power, $E[\boldsymbol{n}\boldsymbol{n}^{\mathsf{H}}] = \sigma_n^2\,\boldsymbol{I}_{n_R}$ with σ_n^2 the noise variance and $E[\boldsymbol{a}\boldsymbol{n}^{\mathsf{H}}] = 0$.

It is well known that the optimal method is the maximum likelihood (ML). However, this method is difficult to use in many applications because it requires a large computation time. In this paper, we propose a faster algorithm whose performances, in terms of bit error rates (BER), are close to the ML performances when the low diversity hypothesis on the channel model is true.

The main limitation to MIMO is the low spatial diversity. This occurs typically for correlated fading MIMO channel [5]. Otherwise, uncorrelated matrix channel containing random entries with similar and non-zero average values could be poorly conditioned, particularly when standard deviations of the matrix entries are not too large with respect to the average values. A typical example is the Rician channel model. This model will be considered throughout the paper.

[1]The superscript $^{\mathsf{H}}$ denotes transpose conjugate and $\boldsymbol{I}_{n_T}$ is the n_T-dimension square identity matrix.

The Rician fading model is used when direct paths (or fixed echoes) between transmitters and receivers are observed [4]. This model approximates a fixed wireless or a slow motion mobile communication system operating in a scattering environment with one direct path. Furthermore, when direct paths between transmitters and receivers become dominant, spatial diversity is low, the standard deviations of the channel matrix entries are small with respect to the average values.

The computational complexity of the ML receiver is due to the fact that the whole set $\mathcal{A}$ of possible vectors $\boldsymbol{a}$ must be evaluated, in order to determine which vector maximizes the ML criterion. The basic idea of the proposed approach is to take profit of the channel model in order to create a partition of the set $\mathcal{A}$ into a number of subsets $\mathcal{A}_i$. Then, we show that the estimation can be divided into two steps: *i)* a first (optimal under low diversity constraint) step, in which we decide which subset is the most likely to contain the transmitted vector. *ii)* a second (optimal) step in which the optimal vector is searched within the selected subset. The gain on computational complexity is due to the fact that the search is restricted to a subset of $\mathcal{A}$. The price to pay is that, due to the split in two steps, the method is not globally optimal. However, as will be shown later, the obtained results are extremely close to the optimal ones, as far as the low diversity hypothesis is true.

II CHANNEL MODEL

Consider a MIMO system with n_R receive and n_T transmit antennas, over which we want to send n_T independent data streams. We assume that the transmission is narrowband, which means that the channel frequency response is constant over the considered bandwidth. It is assumed that channel varies very slowly, so that the random channel matrix $\boldsymbol{H}$ is assumed constant during a burst of symbols.

The uncorrelated MIMO Rician channel model corresponds to the combination of Rayleigh and specular multipath fading [1, 2]:

$$\boldsymbol{H} = \sqrt{\kappa}\, \boldsymbol{H}_m + \sqrt{1-\kappa}\, \widetilde{\boldsymbol{H}} \tag{2}$$

where $\widetilde{\boldsymbol{H}} = [\widetilde{h}_{ij}]$ is a matrix whose entries are independent and identically distributed (i.i.d.), complex normal, zero-mean, and variance equal to one. $\boldsymbol{H}_m = \boldsymbol{\gamma}\boldsymbol{\beta}^{\mathsf{H}}$ is a deterministic rank-one matrix with $\boldsymbol{\gamma} = (\exp(j\theta_1), \ldots, \exp(j\theta_{n_R}))^T$ and $\boldsymbol{\beta} = (\exp(j\theta'_1), \ldots, \exp(j\theta'_{n_T}))^T$ and such that $trace(\boldsymbol{H}_m \boldsymbol{H}^{\mathsf{H}}{}_m) = n_T\, n_R$. The Rician parameter κ (between zero and one) is the fraction of channel energy devoted to specular component. Note that for $\kappa = 0$, specular multipath fading doesn't exist and the MIMO channel is modeled as Rayleigh. For κ close to one, specular multipath fading is dominant, $\boldsymbol{H}$ is poorly conditioned and leads to low spatial channel diversity. Assuming the specular component known at both transmitter and receiver, a phase compensation allows to simplify the model (2) by setting the entries of $\boldsymbol{H}_m$ equal to 1. Without lost of generality, this simplification will be used in the rest of the paper for clarity of presentation of the MLB proposed decoder.

We introduce the following dispersion coefficient ρ defined by:

$$\rho = \sqrt{\frac{1-\kappa}{\kappa}} \tag{3}$$

The dispersion coefficient ρ will be, in the following, a crucial parameter for determination of the domain of validity of the method.

III PROPOSED APPROACH

III.1 Principle of the fast ML-based receiver

Let us note a_Σ the sum of the entries of vector $\boldsymbol{a}$ and r_{av} the average received value:

$$a_\Sigma = \sum_{i=1}^{n_T} a_i \qquad r_{av} = \frac{1}{n_R}\sum_{i=1}^{n_R} r_i \tag{4}$$

where a_i and r_i are the components of the vector $\boldsymbol{a}$ and $\boldsymbol{r}$ respectively. We can also define n_{av} in a similar way. Finally, let us note $\widetilde{\boldsymbol{H}}_{av}$ the $(1 \times n_T)$ vector, the entries of which are the averages of the columns of $\widetilde{\boldsymbol{H}}$.

Now, let us left-multiply Eq. 1 by a $(1 \times n_R)$ vector with entries are equal to $1/n_R$. We get:

$$r_{av} = \sqrt{\kappa}\, a_\Sigma + \sqrt{1-\kappa}\, \widetilde{\boldsymbol{H}}_{av}\boldsymbol{a} + n_{av} \tag{5}$$

Under assumptions:

$$\kappa\, E[|a_\Sigma|^2] = \kappa\, p_0 \quad \gg \quad (1-\kappa)p_0/n_R \tag{6}$$
$$E[|n_{av}|^2] = \sigma_n^2/n_R \quad \gg \quad (1-\kappa)p_0/n_R \tag{7}$$

Eq. 5 can be approximated by:

$$r_{av} \simeq \sqrt{\kappa}\, a_\Sigma + n_{av} \tag{8}$$

Assumption (6) corresponds to the low channel diversity (small values of ρ or values of κ close to one). Assumption (7) links the SNR to the channel diversity: higher the SNR is, smaller ρ must be to satisfy (7).

We can perform a maximum likelihood estimation of a_Σ. Then, assuming that the estimated a_Σ is the true one, the ML search can be restricted to the subset of $\mathcal{A}$ which corresponds to this a_Σ.

III.2 Probabilistic description

In this section we show that the ML estimation of a_Σ from r_{av} in the first step is equivalent to the ML estimation of $\boldsymbol{\alpha} = \sqrt{\kappa}\boldsymbol{H}_m\boldsymbol{a}$ from $\boldsymbol{r}$ under assumptions (6) and (7).

In order to derive the proposed method, we introduce the following definitions:

- Let $\mathcal{A}$ the set of all possible vectors $\boldsymbol{a}$. For a QAM with a constellation of 2^n points the cardinal of $\mathcal{A}$ is $card(\mathcal{A}) = L = 2^{n n_T}$.
- Let $\widetilde{\mathcal{A}}$ the set of possible received symbol vectors through the matrix $\sqrt{\kappa}\boldsymbol{H}_m$ without noise. Elements of $\widetilde{\mathcal{A}}$ are denoted $\boldsymbol{\alpha}^i$ for $i = 1, \ldots, Q$ ($card(\widetilde{\mathcal{A}}) = Q$, see Fig. 1.a).
- Let $\mathcal{A}_i$ the set of symbol vectors $\boldsymbol{a}$ associated to $\boldsymbol{\alpha}^i$ such that $\sqrt{\kappa}\boldsymbol{H}_m\boldsymbol{a} = \boldsymbol{\alpha}^i$ (*i.e.*, $\mathcal{A}_i = \{\boldsymbol{a} \mid \sqrt{\kappa}\boldsymbol{H}_m\boldsymbol{a} = \boldsymbol{\alpha}^i\}$). We notice that each component of $\boldsymbol{\alpha}^i$ is equal to $\sqrt{\kappa}\, a_\Sigma^i$. For each $\boldsymbol{a} \in \mathcal{A}_i$ corresponds a_Σ^i. Each different value of a_Σ is noted a_Σ^i. The cardinal of $\mathcal{A}_i$ is equal to L_i. Sets $\mathcal{A}_i$ for $i = 1, \ldots, Q$ form a partition of $\mathcal{A}$ and then $L = \sum_{i=1}^{Q} L_i$.

The MLB method consists in estimating the vector $\boldsymbol{\alpha}^i$ in order to apply the ML to the set $\mathcal{A}_i$:

1. Determination of the set $\mathcal{A}_i$ among Q possible sets by using an approximate ML estimation of the vector $\boldsymbol{\alpha}^i$. To illustrate this step, Fig.1(a) plots the constellation of a_Σ^i for $n_T = 3$ and a QPSK modulation.
2. Estimation of the symbol vector $\boldsymbol{a}$ by using the ML estimation within the previous estimated set $\mathcal{A}_i$.

Note that the first step is crucial: if the determination of $\mathcal{A}_i$ is correct then estimation of the symbol vector is equivalent to the estimation provided by the ML decoder.

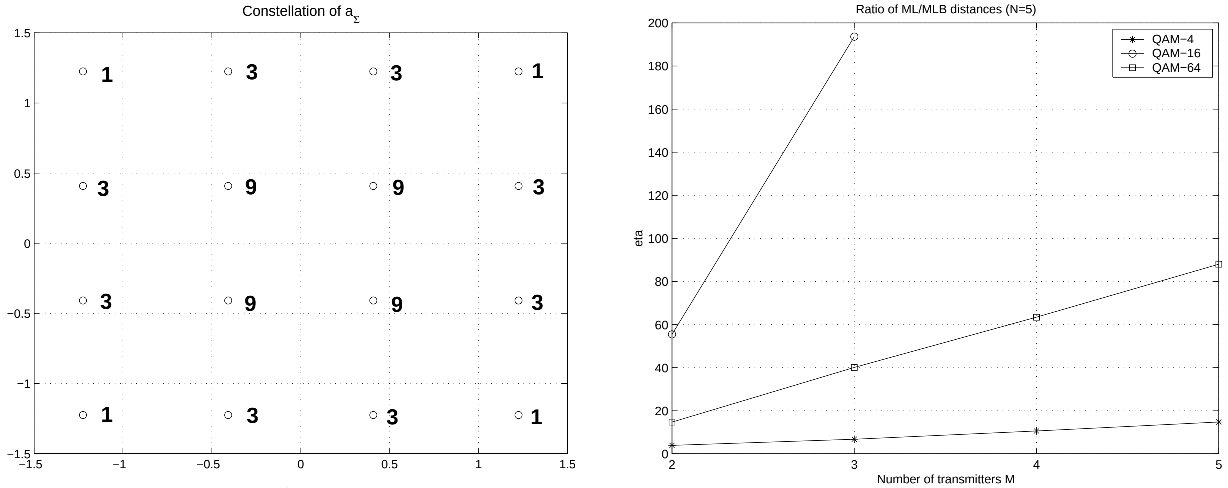
Now, we detail the two steps MLB method. The first step is obtained by computing

$$i = \text{arg}\max_j p(\boldsymbol{r}|\boldsymbol{\alpha}^j) \tag{9}$$

By definition, the knowledge of $\boldsymbol{\alpha}^j$ implies $\boldsymbol{a} \in \mathcal{A}_j$, hence :

$$p(\boldsymbol{r}|\boldsymbol{\alpha}^j) = p(\boldsymbol{r}|\boldsymbol{a} \in \mathcal{A}_j) = \sum_{\boldsymbol{a} \in \mathcal{A}_j} p(\boldsymbol{r}|\boldsymbol{a})P(\boldsymbol{a}|\boldsymbol{a} \in \mathcal{A}_j) = \frac{1}{L_j}\sum_{\boldsymbol{a} \in \mathcal{A}_j} p_{\boldsymbol{n}}(\boldsymbol{r} - \boldsymbol{H}\boldsymbol{a}) \tag{10}$$

where $p_{\boldsymbol{n}}()$ denotes the pdf of $\boldsymbol{n}$.



(a) *(b)*

Figure 1: ***(a)** Constellation a_Σ^i for $n_T = 3$, $n_R = 5$ and a QPSK ($p_0 = 1$). The bold numbers correspond to cardinals of $\mathcal{A}_i$ (*i.e., *L_i) for $i = 1, \dots, 16$. **(b)** Ratio of calculated distances between ML and MLB receivers for $n_R = 5$ and several QAM constellations.*

Under assumption (6) and (7), Eq. (10) can be approximated by:

$$p(\boldsymbol{r}|\boldsymbol{\alpha}^j) \simeq p_{\boldsymbol{n}}(\boldsymbol{r} - \boldsymbol{\alpha}^j) \tag{11}$$

Finally, (9) leads to:

$$i = \arg \min_j \|\boldsymbol{r} - \boldsymbol{\alpha}^j\| \tag{12}$$

which is equivalent to solve:

$$i = \arg \min_j \left| r_{av} - \sqrt{\kappa}\, a_\Sigma^j \right| \tag{13}$$

The second step is performed by computing the ML estimation for $\boldsymbol{a} \in \mathcal{A}_i$ where $\mathcal{A}_i$ is obtained in the first step:

$$\hat{\boldsymbol{a}} = \arg \max_{\boldsymbol{a} \in \mathcal{A}_i} p_{\boldsymbol{n}}(\boldsymbol{r} - \boldsymbol{H}\boldsymbol{a}) \tag{14}$$

which can be simplified as :

$$\hat{\boldsymbol{a}} = \arg \min_{\boldsymbol{a} \in \mathcal{A}_i} \|\boldsymbol{r} - \boldsymbol{H}\boldsymbol{a}\| \tag{15}$$

III.3 Distances computation reduction

In order to illustrate the gain in number of calculated distances between MLB and ML, we compute this ratio η of number of calculated distances for a M-QAM square constellation. For the ML receiver with n_T transmitters, the number of configurations is $L = M^{n_T}$. The symbols a_Σ^i belong to a square constellation of $Q = ((\sqrt{M} - 1)n_T + 1)^2$ elements. The mathematical expectation of L_k is:

$$E[L_k] \quad = \quad \sum_{i=1}^{Q} P(L_i)L_i = \frac{1}{L}\sum_{i=1}^{Q}(L_i)^2 \tag{16}$$

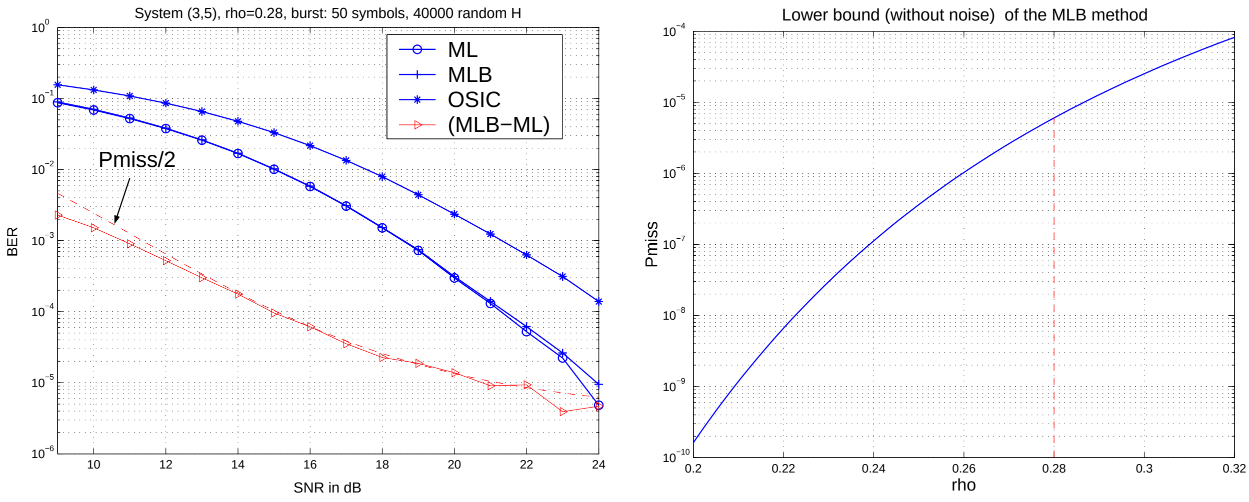
The ratio η in term of calculated distances between the ML and MLB receiver is then

$$\eta = \frac{\overbrace{n_R \times L}^{\text{ML}}}{\underbrace{1 \times Q}_{\text{first step}} + \underbrace{n_R \times E[L_k]}_{\text{2nd step}}} \tag{17}$$

The improvement is represented on Fig.1(b) where η is plotted versus n_T and for 4-QAM, 16 and 64 constellations.

IV SIMULATION RESULTS

We use Monte-Carlo simulations to illustrate performance comparison between ML, MLB and BLAST decoders. Consider a 3×5 spatial MIMO system over which we transmit 3 independent data streams with QPSK modulation. The elements of the uncorrelated MIMO Rician channel $\boldsymbol{H}$ are i.i.d. complex, Gaussian random variables with a dispersion parameter $\rho = 0.28$ (see (2) and (3)). For our simulations, the average BER is obtained by averaging over 40,000 Monte-Carlo simulations runs, and a new $\boldsymbol{H}$ is randomly chosen every 50 symbol vectors.



(a) ***(b)***

Figure 2: ***(a)*** *BER vs signal-to-noise ratio for ML, MLB and OSIC receivers with a QPSK modulation,* $\rho = 0.28$, $n_T = 3$, $n_R = 5$ *and the difference of BER between MLB and ML (legend (MLB-ML)) in comparison with* $P_{miss}/2$. ***(b)*** *Probability* P_{miss} *to choose the bad subset in the first step for the MLB method without AWGN.*

The average signal-to-noise ratio $\overline{SNR}$ is given by:

$$\overline{SNR} = E_H[SNR] = E_H\left[\frac{p_0\, trace(\boldsymbol{H}\boldsymbol{H}^{\mathsf{H}})}{n_T n_R \sigma_n^2}\right] = \frac{p_0}{\sigma_n^2} \tag{18}$$

In Fig.2(a), the average BER is computed for $\rho = 0.28$ and ML, MLB and OSIC decoders. This ordered successive interference cancellation (OSIC) algorithm is used in the V-BLAST system described in [3]. The BER performance of the MLB is very close to the optimal (ML). This excellent result is obtained in spite of the complexity reduction. The ML receiver computes $4^{n_T} \times n_R = 4^3 \times 5 = 320$ distances. The MLB receiver computes in the first step $Q = 16$ distances in order to choose $\mathcal{A}_i$. In the second step the ML search on the previous subset computes in average $E[L_k] \times n_R = 6.25 \times 5 = 31.25$ distances. The ratio η in term of calculated distances is about 6.8.

The sub-optimality of the MLB method, which comes only from the first step, is illustrated in Fig.2(a) where the difference in term of BER between the MLB and the ML is plotted. As shown in this figure, this difference seems to be approximatively equal to $P_{miss}/2$ with P_{miss} the probability to choose the bad subset in the first step of the MLB method. From (5), P_{miss} can be theoretically computed, it corresponds to the symbol error probability of a Q-QAM (constellation of $\sqrt{\kappa}\, a_\Sigma$, see Fig.1(a)) corrupted by the Gaussian random variable ($\widetilde{\boldsymbol{H}}_{av}\boldsymbol{a} + n_{av}$), zero-mean and variance $p_0(1-\kappa)/n_R + \sigma_n^2/n_R$. We obtain[2]:

$$P_{miss} = 2\frac{\sqrt{L}-1}{\sqrt{L}}\,\mathrm{erfc}\sqrt{\frac{3(M-1)}{2}\,SNR_{out}} \quad \text{with} \quad SNR_{out} = \frac{\kappa\, p_0/n_T}{(1-\kappa)p_0/n_R + \sigma_n^2/n_R} \tag{19}$$

Fig.2(b) plots P_{miss} versus the dispersion coefficient ρ (put $\kappa = 1/(1+\rho^2)$ in (19)). We observe a probability of 6×10^{-6} (for $\rho = 0.28$ and $\sigma_n = 0$) to choose the bad subset. This implies that for very

[2]Note that the multiplicative factor is $(\sqrt{L}-1)/\sqrt{L}$ instead of $(\sqrt{Q}-1)/\sqrt{Q}$ because the symbols a_Σ are not equiprobable.

high SNRs the condition (7) doesn't hold any more and the MLB method presents a stage close to $P_{miss}/2$ (the decision $\boldsymbol{\alpha}^i$ in the first step is wrong and $\boldsymbol{a}$ is then chosen in the bad set $\mathcal{A}_i$ even without noise). P_{miss} for $\sigma_n = 0$ corresponds to the lower bound to miss the correct subset in the first step of the MLB method (cf Fig.2(b)). It indicates the validity of the MLB method depending of the dispersion parameter ρ. We conclude that as far as ρ is small (which corresponds to low diversity channel) the method is applicable and gives results very close to the ML ones.

V CONCLUSION

We introduced a decoder algorithm (MLB) for MIMO Rician fading channel with low dispersion. This method is based on the ML estimator but uses the structure of the channel to significatively reduce the computational complexity. The MLB performances in term of BER are similar to the performances of the ML decoder as far as the assumption of low channel's dispersion is satisfied. Otherwise, the ML estimation over the two steps may be replaced by applying the sphere decoding algorithm [6] in order to reduce again the computational complexity.

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