Multi-Layer Perceptrons for Task Visual Servoing in Robotics

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Abstract. A six-degree-of-freedom robot with a CCD-camera mounted on its end-effector is operated in a closed loop control system. The aim is to servo the camera on an object which moves freely in 3D space (translation + rotation), so that the image of the object on the camera screen is always the "reference image", defined by an arbitrary reference position of the object with respect to the camera.
Classical mechanics equations are first studied in order to determine the significant parameters of the problem. Then, a control system based on the use of a MultiLayer Perceptron (MLP) is built. Results and comparisons detailing the behavior of the system in different cases (uniform and non-uniform translations and rotations of the object) are presented, and show the interest of the approach vs. classical methods.

1   Introduction

This paper is concerned with the following visual task servoing problem: a 6 degree-of-freedom robot with a camera-in-hand is made to follow the movement of an object in the 3D space, as shown in Figure (1). An arbitrary picture of the object is first taken by the camera and
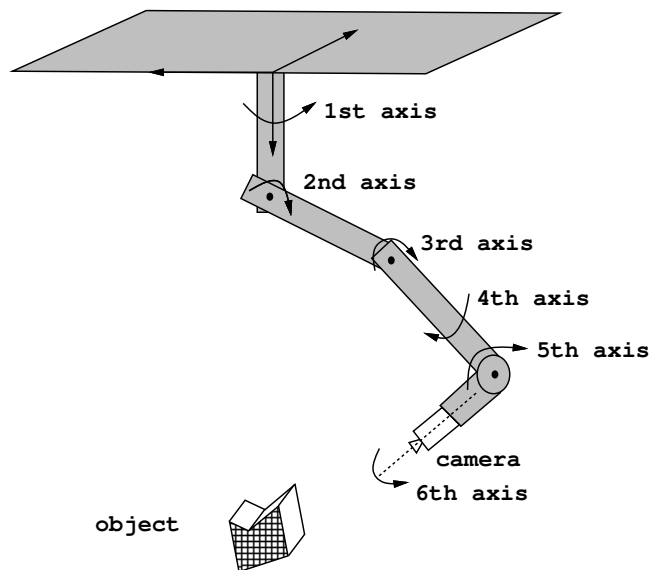

Fig. 1. *A general overview of the system.*

is considered thereafter as the "reference image" associated to this "reference position" of the camera relative to the object. As the object moves, it is desired that the control system send

commands to the robot, so that the relative position between the camera and the object should be as close as possible to its "reference position". This task is said to be servoed visually, in the sense that the error criterion is the discrepancy between the current and the reference image. In order to find a neural solution to this control problem, classical equations are first derived (Section 2). Then, in Section 3, the significant parameters of the problem are detailed, wherefrom a neural solution is investigated (Section 4). Finally, Section 5 provides experimental results and comparisons with previous work. Section 6 draws conclusions on the neural approach.

2   General equations

The pinhole camera model is first presented in order to define which parameters are necessary in the equations of the system. These parameters will then be calculated using the equations of classical mechanics.

2.1   Pinhole camera model

The pinhole camera model can be defined as a convention defining the projection of points in 3D space onto the image plane, following the notation of Figure (2).

The main pinhole camera model equations are:

$$\begin{cases} u = \frac{f}{S_X}\frac{X_C}{Z_C} + u_0 = x + u_0 \\ v = \frac{f}{S_Y}\frac{Y_C}{Z_C} + v_0 = y + v_0 \end{cases}$$

- where (u, v) are the coordinates of point p on the image plane (in pixels),
- (u_0, v_0) are the coordinates of the optical center on the image plane,
- (X_C, Y_C, Z_C) are the coordinates of the object point P_C in the camera coordinate system,
- parameters S_X and S_Y represent the pixel size, and f the focal length.

Derivation of image parameters u and v with respect to time yields:

$$\begin{cases} \frac{du}{dt} = \frac{f}{S_X}\left(\frac{1}{Z_C}\frac{dX_C}{dt} - \frac{X_C}{Z_C^2}\frac{dZ_C}{dt}\right) \\ \frac{dv}{dt} = \frac{f}{S_Y}\left(\frac{1}{Z_C}\frac{dY_C}{dt} - \frac{Y_C}{Z_C^2}\frac{dZ_C}{dt}\right) \end{cases}$$

which is to say, in matrix form:

$$\begin{bmatrix} \frac{du}{dt} \\ \frac{dv}{dt} \end{bmatrix} = \begin{bmatrix} \frac{f}{S_X Z_C} & 0 & -\frac{f X_C}{S_X Z_C^2} \\ 0 & \frac{f}{S_Y Z_C} & -\frac{f Y_C}{S_Y Z_C^2} \end{bmatrix} \begin{bmatrix} \frac{dX_C}{dt} \\ \frac{dY_C}{dt} \\ \frac{dZ_C}{dt} \end{bmatrix}$$

If we call $I = [x\ y]^T$, we have:

$$\frac{dI}{dt} = \begin{bmatrix} \frac{f}{S_X Z_C} & 0 & -\frac{x}{Z_C} \\ 0 & \frac{f}{S_Y Z_C} & -\frac{y}{Z_C} \end{bmatrix} \frac{dP_C}{dt} \tag{1}$$

Therefore $\mathbf{V}_P^C$, the speed of object point P_C in the camera frame, must be evaluated in order to write the system equations. This is where we use classical mechanics equations.
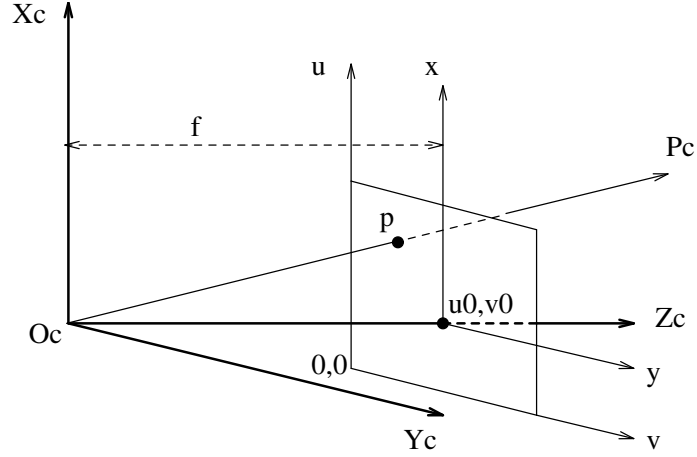
Fig. 2. *Pinhole camera model.*

2.2 Classical mechanics equations

The intantaneous movement of an object can be defined by $C^{\mathcal{E}}(S)$:

$$C^{\mathcal{E}}(S) \equiv \begin{cases} \boldsymbol{\omega}(S/\mathcal{E}) \\ \mathbf{V}_P^{\mathcal{E}} \end{cases}$$

where $\mathcal{E}$ is the coordinate frame, S the object, and P a point of S. $\boldsymbol{\omega}(S/\mathcal{E})$ is the instantaneous rotation speed of S with respect to $\mathcal{E}$, and $\mathbf{V}_P^{\mathcal{E}}$ is the translation speed of point P in $\mathcal{E}$.

Let R_0 be a fixed frame, and R_1 a (possibly) moving frame. The relation describing the composition of movements is:

$$\mathbf{V}_P^{R_0} = \mathbf{V}_P^{R_1} + \mathbf{V}_{P \in R_1}^{R_0}$$

where the last term represents the speed in R_0 of the fixed point of R_1 that coincides with P at the same moment.

If we apply this relation to the fixed frame R of the robot and the camera frame C, we get:

$$\mathbf{V}_P^R = \mathbf{V}_P^C + \mathbf{V}_{P \in C}^R$$

which yields to the value we were looking for:

$$\mathbf{V}_P^C = \mathbf{V}_P^R - \mathbf{V}_{P \in C}^R$$

To aid in the calculation of these two terms, let us write the fundamental relation of kinematics:

$$\forall (P, Q) \in S^2, \mathbf{V}_Q^{\mathcal{E}} = \mathbf{V}_P^{\mathcal{E}} + \boldsymbol{\omega}(S/\mathcal{E}) \wedge \overrightarrow{PQ}$$

and let us use it for $\mathbf{V}_{P \in C}^R$:

$$\mathbf{V}_{P \in C}^R = \mathbf{V}_{O_C}^R + \boldsymbol{\omega}(C/R) \wedge \overrightarrow{O_C P}$$

where O_C is the origin of frame C. In the same way, we can write for a point P of the object:

$$\mathbf{V}_P^R = \mathbf{V}_{O_O}^R + \boldsymbol{\omega}(O/R) \wedge \overrightarrow{O_O P}$$

where O_O is the origin of the object frame.

Finally, in the camera frame, the speed of an object point can be written as:

$$\mathbf{V}_P^C = \mathbf{V}_{O_O}^R + \boldsymbol{\omega}(O/R) \wedge \overrightarrow{O_O P} - \mathbf{V}_{O_C}^R - \boldsymbol{\omega}(C/R) \wedge \overrightarrow{O_C P} \qquad (2)$$

It is interesting to note that the two first terms can be interpreted as the influence of the absolute movement of the object in the camera frame, and the two last terms as the influence of the camera movement.

3 Problem parameters

It is the fusion of the pinhole camera model with the classical mechanics approach that enables writing the problem equations.

3.1 Fundamental equation

The pinhole model described in equation (1) shows that the speed of an object point P_C in the image depends only on the following parameters: its position in the image (x and y), its depth in the camera frame (Z_C), and its speed $\mathbf{V}_P^C$ in the camera frame:

$$\frac{dI}{dt} = \begin{bmatrix} \frac{f}{S_X Z_C} & 0 & -\frac{x}{Z_C} \\ 0 & \frac{f}{S_Y Z_C} & -\frac{y}{Z_C} \end{bmatrix} \mathbf{V}_P^C$$

Remember that we said from Equation (2) that $\mathbf{V}_P^C$ is the sum of two distinct terms: one term which is due to the absolute movement of the object in the fixed frame, and another which is due to the movement of the camera. Hence we can write:

$$\frac{dI}{dt} = \frac{dI_C}{dt} + \frac{dI_O}{dt} \tag{3}$$

where $\frac{dI_C}{dt}$ is the term associated with the object movement in the image due to camera movements only, and $\frac{dI_O}{dt}$ is the term associated with the object movement in the image due to the object's own movements.

Let us explain the value of $\frac{dI_C}{dt}$: the movement of the camera in the absolute (or fixed) frame R produces a movement of the points in the image such that:

$$\frac{dI_C}{dt} = \begin{bmatrix} \frac{f}{S_X Z_C} & 0 & -\frac{x}{Z_C} \\ 0 & \frac{f}{S_Y Z_C} & -\frac{y}{Z_C} \end{bmatrix} \left(-\mathbf{V}_{O_C}^R - \boldsymbol{\omega}(C/R) \wedge \overrightarrow{O_C P} \right) \tag{4}$$

Yet a simple computation gives:

$$\forall (\mathbf{a}, \mathbf{b}), \mathbf{a} \wedge \mathbf{b} = \tilde{a}.\,\mathbf{b}$$

where

$$\tilde{a} = \begin{bmatrix} 0 & -a_Z & a_Y \\ a_Z & 0 & -a_X \\ -a_Z & a_X & 0 \end{bmatrix}$$

therefore we can write

$$\boldsymbol{\omega}(C/R) \wedge \overrightarrow{O_C P} = -\overrightarrow{O_C P} \wedge \boldsymbol{\omega}(C/R)$$
$$= -\widetilde{O_C P}.\,\boldsymbol{\omega}(C/R)$$

Equation (4) may be rewritten in matrix form:

$$\frac{dI_C}{dt} = \begin{bmatrix} \frac{f}{S_X Z_C} & 0 & -\frac{x}{Z_C} \\ 0 & \frac{f}{S_Y Z_C} & -\frac{y}{Z_C} \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 & -Z_C & Y_c \\ 0 & -1 & 0 & Z_C & 0 & -X_C \\ 0 & 0 & -1 & -Y_C & X_C & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{O_C}^R \\ \boldsymbol{\omega}(C/R) \end{bmatrix}$$

which gives, using the definitions of x and y:

$$\frac{dI_C}{dt} = B \begin{bmatrix} \mathbf{V}_{O_C}^R \\ \boldsymbol{\omega}(C/R) \end{bmatrix}$$

where B is the following 6×6 matrix:

$$B = \begin{bmatrix} -\frac{f}{S_X Z_C} & 0 & \frac{x}{Z_C} & \frac{S_Y}{f}xy & -\left(\frac{f}{S_X} + \frac{S_X}{f}x^2\right) & \frac{S_Y}{S_X}y \\ 0 & -\frac{f}{S_Y Z_C} & \frac{y}{Z_C} & \frac{f}{S_Y} + \frac{S_Y}{f}y^2 & -\frac{S_X}{f}xy & -\frac{S_X}{S_Y}x \end{bmatrix} \tag{5}$$

Let us call $\mathbf{u}_C$ the 6×1 vector defined by:

$$\mathbf{u}_C = \begin{bmatrix} \mathbf{V}_{O_C}^R \\ \boldsymbol{\omega}(C/R) \end{bmatrix}$$

and rewrite Equation (3) using this notation:

$$I^\bullet(t) = \frac{dI}{dt} = B\mathbf{u}_C + \frac{dI_O}{dt}$$

At times t and $t-1$ we can write:

$$I^\bullet(t) = B(t)\mathbf{u}_C(t) + I_0^\bullet(t)$$
$$I^\bullet(t-1) = B(t-1)\mathbf{u}_C(t-1) + I_0^\bullet(t-1)$$

If we assume that the time interval between two image acquisitions is short enough, we can make the hypothesis that the movements of points in the image due to object movements are uniform, that is to say $I_0^\bullet(t)$ is constant over time. Therefore, the two preceeding equations can be subtracted and simplified:

$$I^\bullet(t) - I^\bullet(t-1) = B(t)\mathbf{u}_C(t) - B(t-1)\mathbf{u}_C(t-1)$$

Finally, with notations $I^\bullet(t) = \frac{I(t+1)-I(t)}{T}$, and $B = T.B$, we get the fundamental equation of the problem:

$$B(t)\mathbf{u}_C(t) = I(t+1) - 2I(t) + I(t-1) + B(t-1)\mathbf{u}_C(t-1) \tag{6}$$

3.2 Estimators and prediction

The original visual task problem can now be clearly defined, using the introduced notations, as the estimation of the instantaneous speed of the camera $\mathbf{u}_C(t)$ as a function of parameters B, I and $\mathbf{u}_C$ at past instants. The estimated speed vector $\mathbf{u}_C(t)$ is assumed to bring the camera into a position such that, at time $t+1$, the image $I(t+1)$ on the camera screen is the "reference image" I_{ref}. Furthermore, an estimation of $\mathbf{u}_C(t)$ using fundamental Equation (6) requires knowledge of $I(t)$. However, since $\mathbf{u}_C(t)$ is to be calculated *before* time t, and since $I(t)$ is obviously unknown before time t, an estimation $\hat{I}(t)$ has to be made. Here again, an expression of the fundamental Equation (6) at time $t-1$ is needed:

$$\hat{I}(t) = 2I(t-1) - I(t-2) + B(t-1)\mathbf{u}_C(t-1) - B(t-2)\mathbf{u}_C(t-2) \tag{7}$$

Finally, $\hat{I}(t+1) = I_{ref}$ and the estimation of $\hat{I}(t)$ are introduced in (6) to give the estimation of $\mathbf{u}_C(t)$:

$$\mathbf{u}_C(t) = B^+(t)\{I_{ref} - 3I(t-1) + 2I(t-2) - B(t-1)\mathbf{u}_C(t-1) + 2B(t-2)\mathbf{u}_C(t-2)\} \tag{8}$$

Just before time t, parameters $I(t-1), I(t-2), \mathbf{u}_C(t-1)$ and $\mathbf{u}_C(t-2)$ are perfectly known. Moreover, it can be seen from Equation (5) that B(t) depends on parameters $x(t), y(t)$ and $Z_C(t)$ only. $x(t)$ and $y(t)$ are easily deduced from $I(t)$, but the depth $Z_C(t)$ is more difficult to estimate. This is why most articles (see [2] and [3] for example) consider matrix B as a constant, using the fact that, when the camera follows the object nicely, the current image is constantly equal to the "reference image". Therefore B is calculated using the reference image, and its values are kept thereafter. The simplified expression of $\mathbf{u}_C(t)$ is now estimated as:

$$\mathbf{u}_C(t) = \mathrm{B}^+\{I_{ref} - 3I(t-1) + 2I(t-2)\} - \mathbf{u}_C(t-1) + 2\mathbf{u}_C(t-2) \tag{9}$$

In [4] Papanikolopoulos proposed a method to estimate matrix B at each iteration when the distance between the camera and the object is fixed and known, but judged that for full 3D tracking it is still an open problem.

4 A neural solution

We have seen that the classical solution to the visual task problem can be reduced to Equation (9), therefore merely requiring knowledge of $I(t-1), I(t-2), \mathbf{u}_C(t-1)$ and $\mathbf{u}_C(t-2)$. Since these vectors clearly represent the significant parameters, it appears feasible to train (see [6] and [1] for more details concerning the training of neural networks) a neural network (namely a Multi-Layer Perceptron) to learn this control task. Furthermore, having trained the MLP, it would be interesting to compare its performances to those of the classical solution. The main problem here is to build a data base representative of the space of possible movements, such that the network responds correctly to any movement or any series of movements of the object.

4.1 The training set

The training set we want should consist of associated input and output vectors. Obviously, the output vector is a 6×1 vector representing the instantaneous speed $\mathbf{u}_C(t)$. In order to give the neural network the same data as in the classical method, the inputs will consist of a vector containing $I(t-1), I(t-2), \mathbf{u}_C(t-1)$ and $\mathbf{u}_C(t-2)$. This is a $2 \times 6 + 4 \times N$ vector, as I represents the x and y coordinates of N caracteristic points.

To begin with, we should remind that, for any two frames R_0 and R_1, the transformation matrix between R_0 and R_1 is the 4×4 matrix defined by:

$$M_{R_0}^{R_1} = \begin{pmatrix} R & T \\ 0\,0\,0 & 1 \end{pmatrix}$$

where R is the rotation matrix between R_0 and R_1, and T is the translation vector between the origin of R_0 and the origin of R_1. $I(t-2)$ is the projection of the caracteristic points of the object onto the camera at time $t-2$. Therefore to compute $I(t-2)$, we need $M_{co}(t-2)$, the transformation matrix between the object frame and the camera frame (see Figure (3)). It is assumed that $I(t-2)$ and I_{ref} are close, hence $M_{co}(t-2)$ is considered a "noisy" reference-position matrix:

$$M_{co}(t-2) = M_{co\_ref} M_u(t-2)$$

where $M_u(t-2)$ is a random "noise" transformation matrix created by the following process: a uniform rotation and translation "noise" vector $\mathbf{n}_u(t-2)$ is generated, whereupon noise matrix $M_u(t-2)$ is calculated (see Appendix A) using Rodrigues' formula [5].

Once this first position is set, a first translational and rotational object speed $U_0(t-2)$ is randomly chosen. The transformation matrix to the new object frame, $M_{oo'}(t-2)$, is calculated
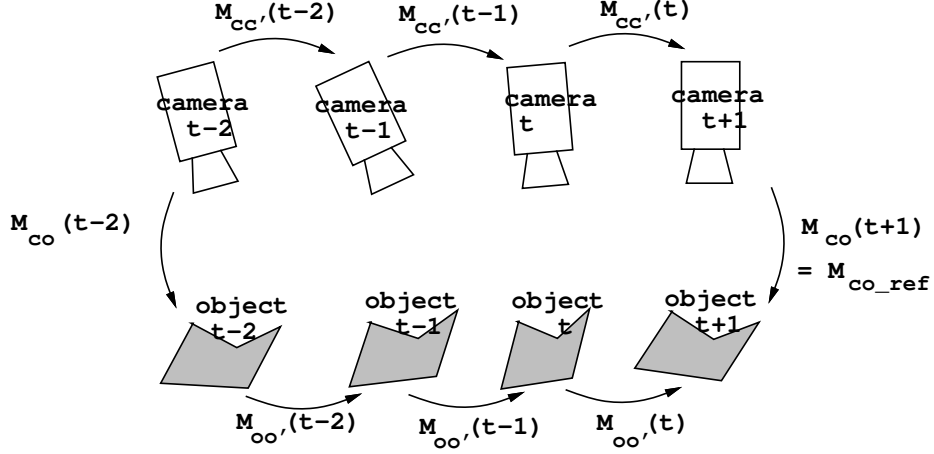
Fig. 3. *Evolution of the relative positions between object and camera through time.*

from $U_0(t-2)$ using Rodrigues' formula as previously. The camera is supposed to follow this movement so as to maintain its reference position with respect to the object: the "ideal" $M_{co}(t-1)$ matrix equals M_{co_ref} and:

$$M_{cc'}^{ideal}(t-2) = M_{co}(t-2)M_{oo'}(t-2)M_{co\_ref}^{-1} \tag{10}$$

The ideal camera speed $U_C(t-2)$ is extracted from $M_{cc'}^{ideal}(t-2)$ using the inverse of Rodrigues' formula. However, since usual movements are rarely perfect, a small random noise vector is added to $U_C(t-2)$: $\mathbf{u}_C(t-2) = U_C(t-2) + \mathbf{n}_C(t-2)$, from which the true $M_{cc'}(t-2)$ matrix is recalculated with Rodrigues' formula. Equation (10) is then modified to let the new relationship between object and camera referentials appear as:

$$M_{co}(t-1) = M_{cc'}^{-1}(t-2)M_{co}(t-2)M_{oo'}(t-2) \tag{11}$$

from which $I(t-1)$ is deduced.

The whole process is rerun for the following iterations (from time $t-1$ to time t). However, this time a constraint is applied, which comes from simple physical considerations: for an object moving at a reasonable speed, and for sufficient sampling over time, it seems probable that the instantaneous speed of the object cannot vary tremendously between two iterations. Therefore $U_0(t-1)$ is assumed to obey:

$$U_0(t-1) = U_0(t-2) + \mathbf{d}_0(t-1)$$

where $\mathbf{d}_0(t-1)$ is a noise vector chosen at random between one tenth of the range used in the choosing $U_0(t-2)$.

The same process is rerun once more (from time t to time $t+1$): a noise vector $\mathbf{d}_0(t)$ is chosen at random and $U_0(t) = U_0(t-1) + \mathbf{d}_0(t)$ yields the object movement matrix $M_{oo'}(t)$. However, since the network is now supposed to be trained to cope perfectly with the various movements of the object, no spurious noise will be added to the "ideal" camera speed extracted from matrix

$$M_{cc'}(t) = M_{co}(t)M_{oo'}(t)M_{co\_ref}^{-1} \tag{12}$$

Therefore $\mathbf{u}_C(t)$ is calculated directly from $M_{cc'}(t)$ by the inversion of Rodrigues' formula. We have now created randomly all the parameters necessary for the training of a network: inputs $I(t-2), I(t-1), \mathbf{u}_C(t-2), \mathbf{u}_C(t-1)$, and the output $\mathbf{u}_C(t)$. The choosing of parameters in random directions ensures the adaptativity of the network for any possible movement of the

object. Furthermore, varying the object speed (speeds at time t, $t-1$ and $t-2$ are different) should provide the possibility to follow an object, even when its speed is changing (non-uniform movement).

4.2 The network

The training set previously described comprises 1000 examples. A two-layer (linear) perceptron is trained on this data: in our experiments, we used an object with $N = 5$ caracteristic points (corners), presented in Figure (4). Therefore $2 \times N = 10$ values are needed to represent every image I, which makes a $2 \times 2 \times N + 2 \times 6 = 32$ input vector for the network. The output $\mathbf{u}_C(t)$
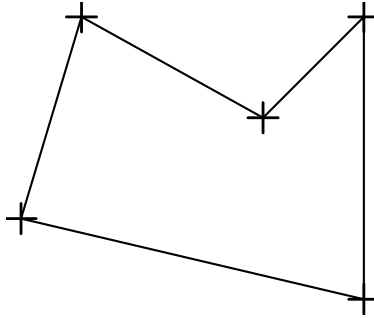

Fig. 4. *Location of the characteristic points of the object.*

comprises 6 units. A 32-input, 6-output, two-layer perceptron with linear activation-function neurons is created and trained on the database until the output error is sufficiently small: learning is considered optimal when the variance of the ouput errors of the network on the training set is smaller than the mean of the output errors of the classical approach [2] [3] on the same data. Once the network has learned, experiments are conducted on various movement cases: linear and non linear translations and rotations.

5 Experiments and discussion

The first part of the experiments is concerned with uniform movement: the object translates or rotates at a constant speed, and the performance criterion is the mean square error between the current position and the reference position of the $N = 5$ caracteristic points over m iterations:

$$e_{MS}(m) = \frac{1}{m} \sum_{n=1}^{m} \sqrt{\frac{1}{N} \|I(n) - I_{ref}\|^2}$$

It is important to note that, in order to present more realistic experiments, the image pixels in $I(n)$ are rounded up to the nearest *even* value (instead of a usual round-up to the nearest integer value). Moreover, a uniform pixel noise between -1 and 1 has been added to each pixel coordinate, so as to be as close as possible to real image processing system defects. In order to show the performances of both the classical method and the neural approach, various object speeds have been tested: Figure (5a) shows the average error in pixels as a function of the translational speed of the object along the X axis (in cm/s). The sampling rate is 4 iterations/second, and each point of the graph represents the average over $m = 200$ iterations. The same experiment was conducted for a uniform rotation of the object. As in the case of translation, it should be noted that the neural network performance is very promising, and even proves to be slightly better than that of the classical approach.
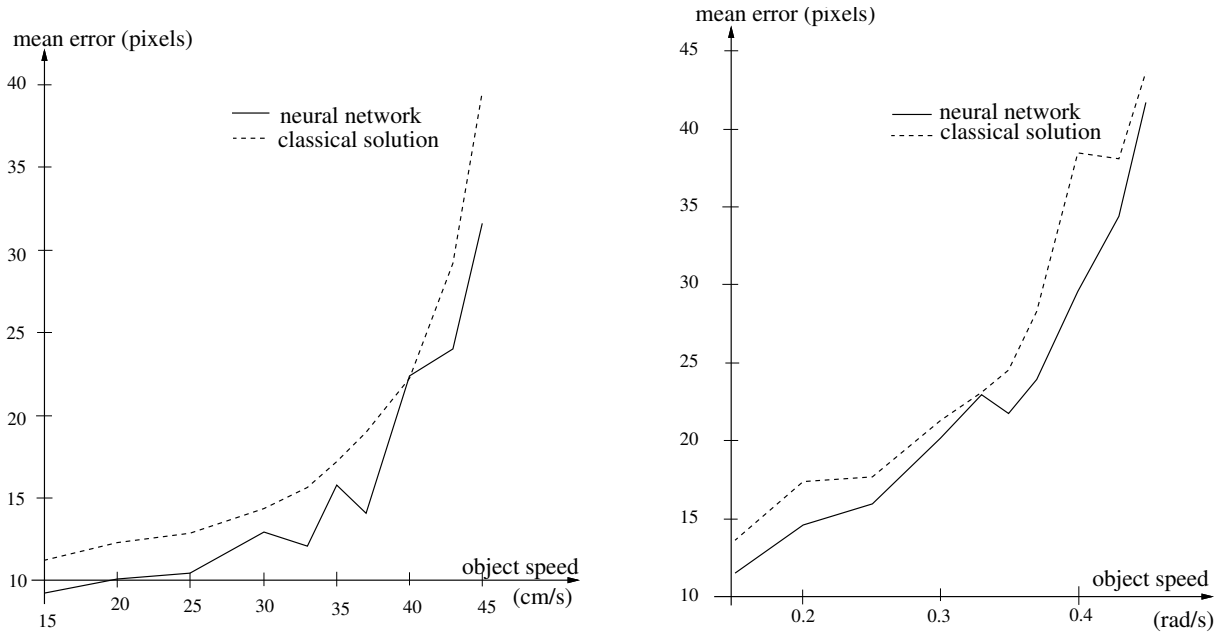
Fig. 5. *Compared performances of the classical method and the neural approach in the case of uniform object speeds: (a) translation, (b) rotation.*

The case of non-uniform object movements is also investigated. In the first non-uniform experiment, the position of the object along the X-axis is a sinusoidal function of the form:

$$x_{pos}(n) = A \; sin(2\pi f n) \tag{13}$$

where n is the number of iterations. The amplitude A of the movement is set to 30cm, while the frequency f varies. For each frequency, a trial is performed for 1000 iterations. The graph showing the average pixel error over 1000 iterations, with respect to the associated frequency, is depicted in Figure (6a). The validity of the neural approach for non-uniform rotations is
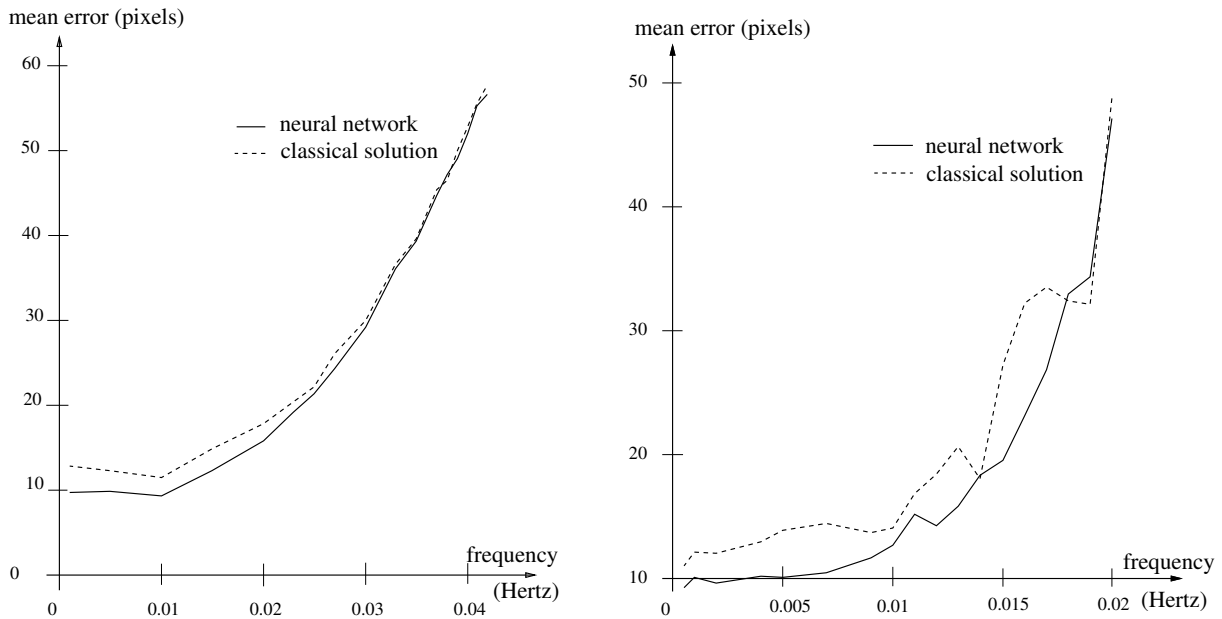

Fig. 6. *Compared performances of the classical method and the neural approach in the case of non-uniform object speeds: (a) sinusoidal translation, (b) sinusoidal rotation.*

also illustrated: sinusoidal rotations around the X-axis are performed (see Figure (6b)) for a rotation angle θ such that:

$$\theta(n) = \frac{\pi}{3} sin(2\pi f n) \tag{14}$$

Sinusoidal (therefore non-uniform) movements appear to prove that the neural approach can handle the problem in a satisfactory way. As is the case for uniform movements, the neural network can compete positively with the classical method, therefore showing its interesting and wide capabilities.

6 Conclusions and perspectives

In a visual task-reference problem, a robot with a camera mounted on its end-effector is used to follow the movement of an object in 3D space: the link between the camera and the object is desired to be rigid. After having identified the significant parameters, it is possible to design and train a two-layer perceptron dedicated to this control problem. Various experiments, including highly non-uniform movements for the object, have been presented and compared with the classical method, showing the interest of the approach. Furthermore, one advantage is the ease with which the neural approach may be extended to non-linear loops, and/or to control loops taking more information into account: such extensions merely require extra or new training.

Further studies include the extension to three-layer (non linear) perceptrons used in a particular way: once the two-layer perceptron has learned, a three-layer perceptron is trained to correct the two-layer MLP, that is to say its inputs are the same, but its outputs are the difference between the correct answer and the answer provided by the two-layer MLP. This way the three-layer MLP can only improve the performances of the two-layer MLP, and the learning phase is much quicker than a direct three-layer learning. Other extensions are planned using more than two iterations in the prediction process, e.g. $I(t-3)$ and $\mathbf{u}_C(t-3)$ might be added to the network inputs.

Acknowledgements

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Appendix A

For a unit time the transformation matrix M_u associated to the 6×1 rotation and translation speed vector $\mathbf{u} = [\mathbf{t}, \boldsymbol{\omega}]^T$ is

$$M_u = \begin{pmatrix} R_u & t \\ 0\,0\,0 & 1 \end{pmatrix}$$

R_u is calculated with the aid of Rodrigues' formula:

$$R_u = I + \sin\theta U + (1 - \cos\theta)U^2$$

where I is the 3×3 identity matrix, θ is the norm (or length) of rotation vector $\boldsymbol{\omega}$, and U is the 3×3 antisymmetric matrix such that:

$$U = \begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix}$$

where $\mathbf{v} = [a\ b\ c]^T$ is the unit vector collinear to $\boldsymbol{\omega}$: $\mathbf{v} = \boldsymbol{\omega}/\|\boldsymbol{\omega}\|$.