Performance analysis of the maximum-SNR design in Rayleigh fading MIMO channels

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PERFORMANCE ANALYSIS OF THE MAXIMUM-SNR DESIGN IN RAYLEIGH FADING MIMO CHANNELS

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Abstract - The performance of the maximum-SNR design for multiple-input multiple-output (MIMO) systems is analyzed. An average symbol error probability (SEP), expressed in closed form, is derived assuming both i.i.d. Rayleigh fading MIMO channel and coherently detected M-ary PSK/QAM. This form highlights a *modulation gain*, which permits comparison of SEP performances between different modulations schemes. The maximum diversity advantage can be retrieved by using a Taylor series expansion around infinity from the exact SEP. The performance of the max-SNR MIMO systems is compared by changing the transmit and receive diversity, modulations and constellation size.

Keywords - MIMO, Maximum-SNR, Maximum eigenvalue p.d.f., Performances, Modulation gain, Diversity order.

I. INTRODUCTION

In the recent years, multiple-input multiple-output (MIMO) systems have known an increasingly fast development thanks to rich scattering wireless channels [1][2]. The reliability of the transmission can be improved by choosing a communication strategy that can withstand the multipath propagation-caused fading dips in the received signal-to-noise ratio (SNR). Under the assumption of channel state information (CSI) known both at the transmitter and the receiver, an efficient solution, denoted maximum-SNR (max-SNR), can be used to improve transmission robustness. This scheme is, sometimes, referred to as beamforming or MIMO maximum ratio combiner (MRC) [3]. This solution consists in transmitting the signals along the strongest direction of the channel, i.e. the direction of the eigenvector corresponding to the largest eigenvalue of[1] $\mathbf{W} = \mathbf{H}\mathbf{H}^*$ where $\mathbf{H} = [h_{ij}]_{i,j=1}^{n_R,n_T}$ is the $n_R \times n_T$ channel matrix with h_{ij} the gain factor from the j^{th} transmit antenna to the i^{th} receive antenna, n_T and n_R are the number of transmit and receive antennas, respectively.

The input-output relation is then:

$$y = \sqrt{P_0}\boldsymbol{w}_R^*\mathbf{H}\boldsymbol{w}_T s + \boldsymbol{w}_R^*\boldsymbol{n} \tag{1}$$

where $\boldsymbol{w}_T$ and $\boldsymbol{w}_R$ are the transmit and receive weight vectors, s is the transmit symbol with $E[|s|^2] = 1$, P_0 is the average power of the received signal at each receive antenna and $\boldsymbol{n}$ is the complex circular Gaussian noise vector with covariance matrix $\mathbf{R}_n = E[\boldsymbol{n}\boldsymbol{n}^*] = \sigma^2\mathbf{I}_{n_R}$. The weight vectors, $\boldsymbol{w}_T$ and $\boldsymbol{w}_R$, are respectively the principal right and left singular vectors to the matrix $\mathbf{H}$, so that the channel matrix can be seen as only the largest singular value $\sigma_{\max} = \sqrt{\lambda_{\max}}$ of $\mathbf{H}$; the receiver SNR is thus maximized [3] and given by $\gamma_0 = P_0\lambda_{\max}/\sigma^2$. The equivalent input-output relation (1) becomes:

$$y = \sqrt{P_0\,\lambda_{\max}}\,s + n \tag{2}$$

where $n = \boldsymbol{w}_R^*\boldsymbol{n}$ is a complex circular Gaussian random variable (RV) with $E[|n|^2] = \sigma^2$.

In the present paper, we base our work on Dighe *et al.* [4] and Kang *et al.* [5] to evaluate the performance of the max-SNR system theoretically in term of symbol error probability (SEP) by assuming an i.i.d. Rayleigh fading channel; i.e. the channel gains between any pair of antennas are supposed to be i.i.d. with zero-mean complex circular Gaussian RV and unit variance. Indeed, SEP is attributable to the determination of the marginal probability density function (p.d.f.) of the maximal eigenvalue $\lambda_{\max}$ of the Wishart matrix $\mathbf{W}$ [5], [6]. Then, we determine from the SEP the parameters able to provide information about the performances of the max-SNR systems: the diversity order and a *modulation gain*.

The next Section of this paper deals with the determination of the p.d.f. of the maximal eigenvalue. Section III describes the calculation of the average SEP expressed in closed form to get an exploitable and convenient analytical formula. It also shows that this procedure yields a *modulation gain* useful for SEP comparisons. From this theoretical SEP and by using Taylor series expansion (t.s.e.), Section IV provides the diversity order. Performances of the max-SNR MIMO systems are also analyzed and discussed in Section V before concluding.

II. PROBABILITY DENSITY FUNCTION OF THE LARGEST EIGENVALUE OF THE WISHART MATRIX

The first part of this paper is about the determination of the p.d.f. of the largest eigenvalue to the Wishart matrix in a closed form for a given arbitrary (n_T, n_R) system.

According to [5], in the central and i.i.d. cases, the cumulative distribution function (c.d.f.) of the largest eigenvalue λ_{max} of the Wishart matrix $\mathbf{W}$ is expressed as follows:

$$F_{\lambda_{\max}}(u) = P\left(\lambda_{\max} < u\right) = \alpha\left|\Psi_c(u)\right| \tag{3}$$

[1]The symbol $*$ denotes transpose conjugate.

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Table 1
Coefficients $c_{n,i}$ of $\phi_n(x)$

(n_T, n_R)	n	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}	x^{12}
$(2,2)$	1 2	2 -2	-2	1										
$(2,3)$	1 2		3 -3	-2 -1	1/2									
$(3,3)$	1 2 3	3 -6 3	-6 6	6 -3	-2 -1	1/4 -1/2								
$(4,4)$	1 2 3 4	4 -12 12 -4	-12 24 -12	18 -24 6	-34/3 8/3 14/3	7/2 -4/3 23/6	-1/2 4/3 4/6	1/36 -4/9 1/12	1/18	-1/72				
$(4,5)$	1 2 3 4		10 -30 30 -10	-20 30 0 -10	35/2 -15/2 -15/2 -5/2	-22/3 -11/2 3 -1/6	19/12 11/6 19/12	-1/6 -5/6 1	1/144 13/36 43/144	-1/12 1/24	1/96 1/288	-1/864		
$(5,5)$	1 2 3 4 5	5 -20 30 -20 5	-20 60 -60 20	40 -90 60 -10	-110/3 40 10 -40/3	215/12 -45/4 55/4 -185/12	-29/6 33/4 -7 -77/12	13/18 -139/24 2/3 -103/72	-1/18 17/8 1/6 -11/72	1/576 -59/96 9/64 -1/144	115/864 35/864	-11/576 11/864	1/576 1/864	-1/10368 1/6912

where $[\Psi_c(u)]_{i,j} = \Gamma_u(n_S + i + j - 1)$ with $n_S = \max(n_T, n_R) - m$, $m = \min(n_T, n_R)$, $|\cdot|$ denotes the determinant and $\Gamma_u(p)$ the incomplete Gamma function

$$\Gamma_u(p) = \frac{1}{\Gamma(p)} \int_0^u t^{p-1} e^{-t} dt = 1 - e^{-u} \sum_{k=0}^{p-1} u^k / k! \quad (4)$$

with $\Gamma(p)$ the complete Gamma function ($\Gamma(p) = (p-1)!$ for p a positive integer) and the normalization coefficient is given by $\alpha = \frac{1}{|\Psi_c(\infty)|} = 1/\prod_{k=1}^{m} \Gamma(n_S + m - k + 1)\Gamma(m - k + 1)$.

The use of the classical formula about the derivative, i.e. $\frac{d}{du}|\boldsymbol{A}(u)| = |\boldsymbol{A}(u)| \operatorname{trace}\left(\boldsymbol{A}^{-1}(u)\frac{d}{dt}\boldsymbol{A}(u)\right)$, gives the p.d.f. of $\lambda_{\max}$ as shown in [5]

$$p_{\lambda_{max}}(u) = \alpha\, |\Psi_c(u)| \operatorname{trace}(\Psi_c^{-1}(u)\Phi_c(u)) \quad (5)$$

where $[\Phi_c(u)]_{i,j} = u^{n_S+i+j-2} e^{-u}$ with $i,j = 1, \ldots, m$. This form allows us to compute efficiently the p.d.f. of the maximal eigenvalue by using a symbolic programming language (e.g. Maple) for an arbitrary given (n_T, n_R) system.

An alternative closed form of the p.d.f. $p_{\lambda_{max}}(u)$ is proposed in the following. By using the definition of the determinant and the Hankel matrix structure of $\Psi_c(u)$, relation (3) can be re-expressed as follows [7] (Burel has expressed the c.d.f. of the smallest eigenvalue):

$$F_{\lambda_{\max}}(u) = \alpha \sum_{\boldsymbol{k} \in P_m} \varepsilon(\boldsymbol{k}) \prod_{i=1}^{m} \hat{\Gamma}_u\left(n_S + k_i + i\right) \quad (6)$$

where $\hat{\Gamma}_u(p) = \Gamma_u(p)\Gamma(p)$ and P_m is the set of all the permutations of $[0, 1, \ldots, m-1]$, $\boldsymbol{k} = [k_1, k_2, \ldots, k_m]$ is an element of P_m, and $\varepsilon(\boldsymbol{k})$ the permutation signature.

Calculation of $P(\lambda_{\max} < u)$ derivative over u gives the p.d.f. of $\lambda_{\max}$ in a new closed form, and then leads to the following expression:

$$\begin{aligned} p_{\lambda_{\max}}(u) = \alpha\, e^{-u} \sum_{\boldsymbol{k} \in P_m} \varepsilon(\boldsymbol{k}) \sum_{j=1}^{m} u^{n_S + k_j + j - 1} \\ \times \prod_{i=1, i \neq j}^{m} \hat{\Gamma}_u(n_S + k_i + i). \end{aligned} \quad (7)$$

Eq. (7) gives a general closed expression of the p.d.f. of $\lambda_{\max}$. By using (4), we hereabove proved that the p.d.f. (7) can be written in the form of:

$$p_{\lambda_{\max}}(u) = \sum_{n=1}^{m} \phi_n(u) e^{-nu} \quad (8)$$

where $\phi_n(u) = \sum_{i=0}^{D_n} c_{n,i} u^i$ is a polynomial with $c_{n,i}$ the i^{th} coefficient of the n^{th} polynomial and D_n is the maximal degree of the n^{th} polynomial. Eq. (8) has already been observed by [8] and [4]. One should note that the polynomials $\phi_n(u)$ have not a literal expression (intractable problem: the evaluation of (7), for a given (n_T, n_R), exhibits many cancellation of terms); however, it is not necessary because the polynomials in (8) are directly extracted from (7).

These coefficients of $\phi_n(u)$ are given in Table 1 for some (n_T, n_R) couples. From (7), let us find the possible highest polynomial degree given by $Q = 2\sum_{i=0}^{m-1} k_i + m\, n_S = m(m - 1 + n_S)$. However, the summation over $\boldsymbol{k}$ allows numerous simplifications, and then the effective maximum degree D_n is less than Q as shown in Table 1. One can verify that D_n is given by $(n_T + n_R)n + (n+1)n$ and the smallest degree is n_S [4].

Note that, whenever the numbers of antennas, x and y, are fixed, the two systems (x, y) and (y, x) are equivalent because the statistical distribution of λ_{max} depends only on $m = \min(x, y)$ and $n_S = \max(x, y) - \min(x, y)$.

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III. SEP IN A RAYLEIGH FADING CHANNEL

The average SEP of coherent BPSK, M-ary QAM and M-PSK in a Rayleigh fading channel is given by

$$\bar{P}_e = \int_0^{\infty} \alpha_M \operatorname{erfc}\left(\sqrt{\beta_M \frac{uP_0}{\sigma^2}}\right) p_{\lambda_{\max}}(u) du \qquad (9)$$

where $\alpha_M = 1/2$, $\beta_M = 1$ for BPSK and $\alpha_M = 2\left(1 - \frac{1}{\sqrt{M}}\right)$, $\beta_M = \frac{3}{2(M-1)}$ for M-ary squared QAM and $\alpha_M = 1$, $\beta_M = \sin^2(\pi/M)$ for M-PSK. Except for the BPSK case, note that (9) is an approximation of the exact average SEP. But, the exact average SEP must be evaluated by using Craig's formula [4]. However (9) is a commonly used approximation which gives a very tightly upper bound at high SNR [9, Sec. 8.1.1]. Furthermore, (9) will allow to express in a unified framework the average SEP for an M-ary modulation, which is useful for performance comparisons.

Section II showed that $p_{\lambda_{\max}}(u)$ is expressed as sums of polynomials multiplied by exponential. Thus, the average SEP can be derived by using (8) and (9) to get

$$\bar{P}_e = \sum_{n=1}^{m}\sum_{i=0}^{D_n} P_{e_{n,i}} \qquad (10)$$

$$\text{with } P_{e_{n,i}} = b_{n,i} \int_0^{\infty} \alpha_M \operatorname{erfc}\left(\sqrt{\beta_M \frac{uP_0}{\sigma^2}}\right) \times \frac{n^{i+1}}{i!} u^i e^{-nu} du \qquad (11)$$

$$\text{and } b_{n,i} = \frac{c_{n,i}\, i!}{n^{i+1}}. \qquad (12)$$

According to currently admitted results about the performance of the MRC [10, chap.7], relation (11) becomes:

$$P_{e_{n,i}} = \alpha_M b_{n,i} \left[1 - \sqrt{\frac{\frac{\beta_M P_0}{\sigma^2}}{\frac{\beta_M P_0}{\sigma^2} + n}} \times \left(1 + \sum_{k=1}^{i} \frac{\frac{(2k-1)!!}{(2k)!!}}{\left(1 + \frac{\beta_M P_0/\sigma^2}{n}\right)^k}\right)\right] \qquad (13)$$

with $(2k-1)!! = 1 \times 3 \times \ldots \times (2k-1) = (2k)!/(k!\,2^k)$ and $(2k)!! = 2 \times 4 \times \ldots \times (2k) = k!\,2^k$.

By using the normalization condition, i.e. $\int p_{\lambda_{\max}}(u)du = 1$, (8) gives the following condition $\sum_{n=1}^{m}\sum_{i=0}^{D_n} b_{n,i} = 1$. The average SEP expressed with the analytical expression issued from (10) and (13) is

$$\bar{P}_e = \alpha_M \left[1 - \sum_{n=1}^{m} \sqrt{\frac{\frac{\beta_M P_0}{\sigma^2}}{\frac{\beta_M P_0}{\sigma^2} + n}}\, \varphi_n\left(\frac{\beta_M P_0}{\sigma^2}\right)\right] \qquad (14)$$

where $\varphi_n(x)$ is a rational polynomial given by

$$\varphi_n(x) = \sum_{i=0}^{D_n} b_{n,i} \left(1 + \sum_{k=1}^{i} \frac{\frac{(2k-1)!!}{(2k)!!}}{\left(1 + \frac{x}{n}\right)^k}\right). \qquad (15)$$

Eq. (14) for $\alpha_M = 1/2$ and $\beta_M = 1$ is equivalent to the BPSK closed-form presented in [8], [4]. The analytical form of the polynomial $\varphi_n(x)$ was not provided in [8]; only the rational polynomial was given for some (n_T, n_R). On the other hand, our study demonstrates that the generalization to M-ary modulations is straightforward by using a unified framework. $\bar{P}_e$ directly depends on the input SNR P_0/σ^2, then (14) allows one to define a *modulation gain* G_M in dB for different M-ary modulations schemes (M-PSK, M-QAM) as follows:

$$G_M = 10 \log_{10}\left(\frac{\beta_{M_1}}{\beta_{M_2}}\right) \qquad (16)$$

where β_{M_1} and β_{M_2} are the β_M factor of the modulations 1 and 2, respectively. At high SNR, this gain shifts the $\bar{P}_e$ curves for a fixed SEP as verified in section V. This result is useful to easily predict the relative performance of the max-SNR design further to change in modulation and/or constellation size.

IV. DIVERSITY ADVANTAGE

The diversity order of an (n_T, n_R) max-SNR system over i.i.d. Rayleigh channel is expected to be equal to $n_T \times n_R$. Traditionally, this result is based on the fact that the lower bound of the maximum output SNR γ_0 is given by $\|H\|_F^2 \times P_0/(\sigma^2 n_T)$ and that, for an i.i.d Rayleigh channel $\|H\|_F^2$ is a χ^2-distributed RV with $2 \times n_T \times n_R$ degrees of freedom [11].

The approach proposed here is to take the Chernoff bound ($\operatorname{erfc}(x) = e^{-x^2}$ for $x \gg 1$) in (11). It then leads to the upper bound of the average SEP

$$\bar{P}_{e_{ub}} = \alpha_M \sum_{n=1}^{m}\sum_{i=0}^{D_n} \frac{b_{n,i}}{\left(1 + \frac{\beta_M P_0}{n\sigma^2}\right)^{i+1}}. \qquad (17)$$

Let us note that the t.s.e. of (17) corresponds to the sum of elementary t.s.e. $\frac{1}{\left(1+\frac{x}{n}\right)^{i+1}}$. The t.s.e. to order d of (17) is then:

$$\bar{P}_{e_{tse}}(x) = \alpha_M \sum_{k=1}^{d} K_k \left(\frac{1}{x}\right)^k + \mathcal{O}\left(\left(\frac{1}{x}\right)^{d+1}\right) \qquad (18)$$

where:

$$K_k = \sum_{n=1}^{m}\sum_{i=0}^{k-1} (-1)^{k+i+1} b_{n,i} \begin{pmatrix} k-1 \\ i \end{pmatrix} n^k. \qquad (19)$$

Eq. (19) allows us to compute K_k and verify that $K_k = 0$ for $k = 1$ to $n_T n_R - 1$. One should note that the coefficients $b_{n,i}$ are needed to compute (19); but the lack of literal form for expressing them prevents one from formally proving K_k cancellation. But, it highlighted that in the high SNR limit (i.e. $P_0/\sigma^2 \gg 1$), by using a Taylor series expansion along the real axis around infinity of (17), the asymptotic equation of the SEP is given by:

$$\bar{P}_{e_{asp}} = \alpha_M K_{n_T n_R} \left(\frac{\beta_M P_0}{\sigma^2}\right)^{-n_T n_R}. \qquad (20)$$

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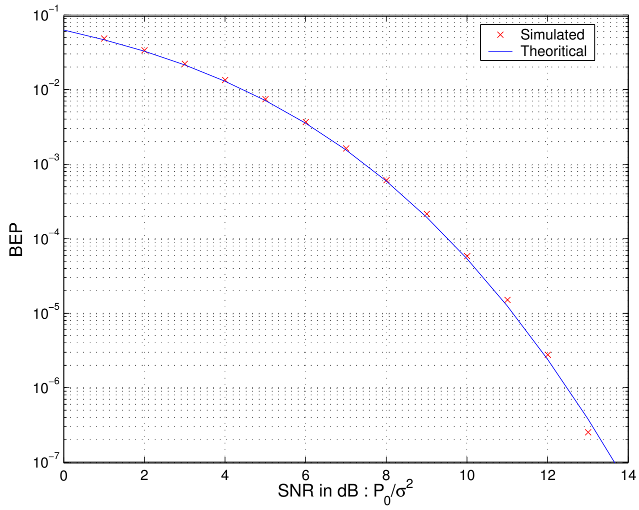

Fig. 1
Simulation and Theoretical BEP comparison for system (4,4) with a QAM-16

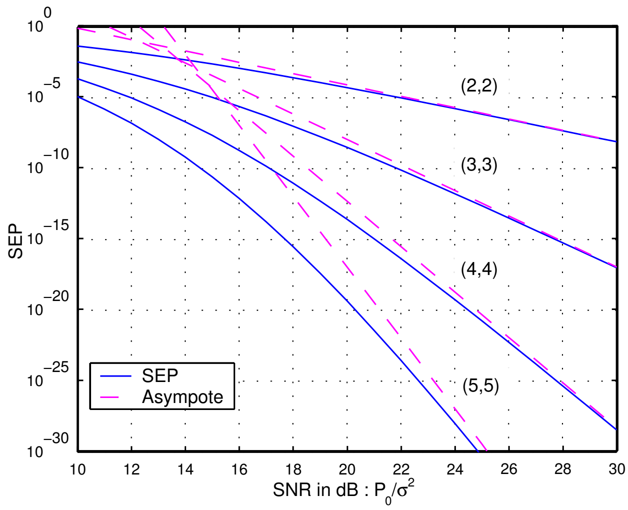

Fig. 2
Performance comparisons (SEP and asymptote) for (2,2) to (5,5) systems with a 16-QAM

It is worth noting that *i)* the *modulation gain* G_M, which depends on β_M, appears clearly in (20) and that *ii)* the maximum diversity advantage can be also retrieved from the exact SEP (14), but with a corresponding factor $K_{n_T n_R}$ more complex than (19).

V. Performance Analysis

Figure 1 shows that (14) fits perfectly the bit error rate (BER) curves obtain by simulation (50,000 random $\mathbf{H}$ are used for each 16-QAM symbol), and thus validate our method. Note that $\bar{P}_e$ is divided by $\log_2(16)$ to obtain bit error probability (BEP) performance. Different (n_T, n_R) systems with several M-ary modulations are tested and confirm the good agreement.

As expected, Figure 2 demonstrates that the higher the diversity order is, the greater the asymptote slope is. On the other hand, the asymptotes are tangent for very high SNR. In addition, for the $(5,5)$ system, asymptote and curve are close when SEP is less than 10^{-30}. This figure highlights a significant performance enhancement when passing from one system to the next one only by the addition of one transmit and one receive antennas at a time, which reveals the interest of transmit and receive diversity.

Figure 3 compares the SEP obtained for 16-QAM and for different numbers of antennas to illustrate the compromise between the diversity order and the total number of antennas. It shows that, for a given total number of antennas, the higher the diversity order is, the better the obtained performance is. For instance, with 8 antennas, the best configuration is (4,4). In other respects, the possibility to coherently combine more signals at the receiver leads to the maximum combining gain. For a given diversity order, the best final performance is obtained with the highest possible number of antennas. With a diversity order of 16, the systems performances are ranked as $(1,16) > (2,8) > (4,4)$ which correspond to $17 > 10 > 8$ as total number of antennas. However, for a pre-set total number of antennas, in term of SEP, a balanced distribution of antennas between transmitter and receiver has to be privileged.

Figure 4 plots SEP for a (8,8) system with different modulations and constellation sizes. It shows that the gap between the curves at moderate and high SNR is close to the *modulation gain* (16). For example: *i)* for a modulation between 16-QAM and 64-QAM, this figure evidences a *modulation gain* of about $6.25\ dB$, and relation (16) gives $6.23\ dB$; *ii)* for a different modulation, the *modulation gain* is $5.75\ dB$ between 4-QAM and 8-PSK instead of $5.33\ dB$. From (20), it is worth noting that, higher the SNR is, more exact the parameter G_M is. However, G_M remains a valid comparison factor even for a large system (e.g. (8,8)), which has the exact SEP close to its asymptote at very high SNR.

VI. Conclusion

We investigated the max-SNR MIMO system by introducing an analytical form of the probability density function of the highest eigenvalue of the Wishart matrix for i.i.d. Rayleigh fading MIMO channel. This representation proved its efficiency to provide the theoretical SEP. We showed that this SEP can be easily applied to M-PSK and M-QAM through modulation parameters (α_M and β_M). We also defined a *modulation gain* to assess any enhancement in performance between two different modulations and/or constellation sizes with the same (n_T, n_R). The *modulation gain*, usually used for Gaussian channel, remains a relevant

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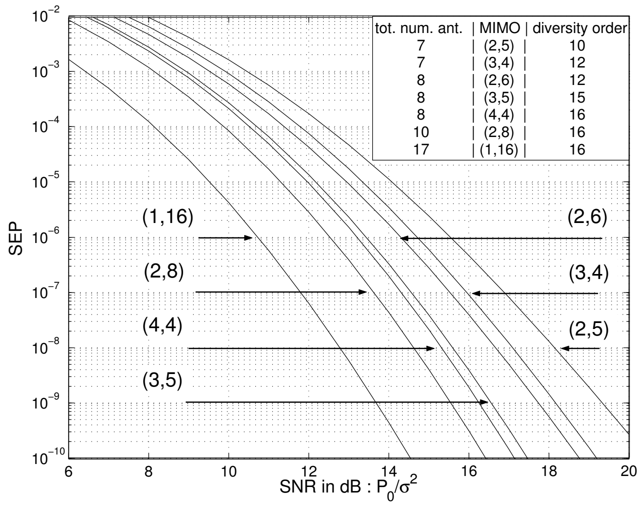

Fig. 3
Performance comparisons for different numbers of antennas with a 16-QAM

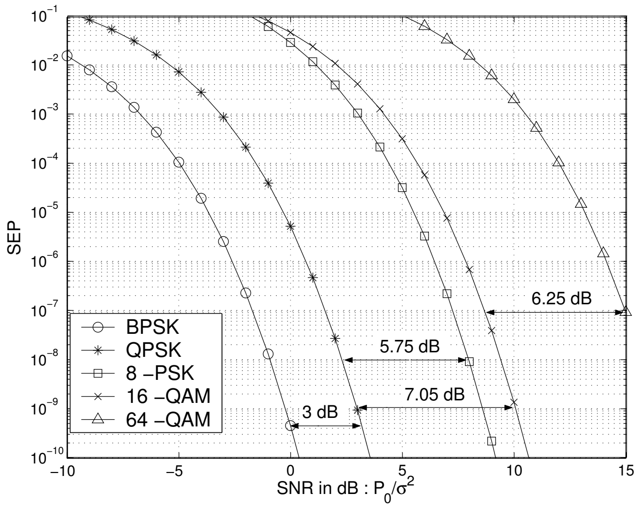

Fig. 4
Performance comparisons for different modulations and $n_T = n_R = 8$

parameter to evaluate and compare performances for max-SNR MIMO systems operating in a Rayleigh fading channel. Moreover, we highlighted directly from the theoretical SEP expression that the diversity order of max-SNR MIMO systems is equal to $n_T \times n_R$. These results are very convenient to predict and compare performance about system configuration: number of antennas, diversity order, modulation and constellation size.

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