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1 Introduction

Most real-world problems require simultaneous optimization of multiple competing objectives. Hence, to better capture the real structure of a given problem, the use of the multiobjective approach becomes inevitable and allows to provide better solutions to the posed problem. In contrast to a single criteria optimization problem, the solution to a multi-objective optimization problem is usually a combination of solutions which form a compromise between different objectives. These solutions are also known as Pareto-Optimal Solutions or efficient solutions. Therefore, the goal of Multiobjective Optimization, also called Multicriteria Optimization, consists in obtaining optimal Pareto solutions and, consequently, in knowing all possible compromises between the objectives.

Over the past decade, researchers have incorporated the idea of game theory and the notion of balance. However, not all methods can be applied to solving real problems with considerable difficulties. Here is a brief overview of methods that incorporate ideas from the theory of cooperative and/or non-cooperative games.

- [5] developed a multi-objective model of integer linear programming, designed to optimize supply chain planning using game theory.

- A multiobjective categorization method based on the theory of Markov games and processes is proposed in [1].

- An application of the theory of cooperative and non-cooperative games was presented in [3].

- In [4], the authors propose a realistic representation of a decision maker's behavior by synthesizing games and purpose programming in a single framework.

- In [2], the authors introduced an iterative method for solving linear multi-objective programming with an arbitrary number of decision makers. The method is based on the principles of game theory.

Solving multi-objective linear programming problems has received quite some attention from the scientific community. Several researchers have opted for metaheuristics resolution. In this

work, we propose a new method of solving linear bi-objective programming problems. This method finds the Pareto front of a linear programming bi-objective problem by performing a transformation into non-cooperative game theory problem. Then, the problem becames a Nash equilibrium search; we will be able to find the two points that form the limitation of the front defining all efficient solutions to the original problem. Computational tests are performed to show the effectiveness of the proposed technique.

2 Presentation of the method

In this section, we propose a new method based on game theory concepts to solve a linear bi-objective programming problem. This method is based on two phases:

2.1 First phase: Enumeration of feasible basic solutions

This is the oldest method for solving linear programming problems; it is based on the theorem that the optimal solution is an extreme point of the polyhedron defined by the set of feasible solutions. It is summarized by the following steps:

- Determine all extreme points.

- Evaluate the objective function at each of these points.

- Compare the values obtained.

However, this method is very slow for large problems.

2.2 Second phase: Transformation of the bi-objective problem into a game theory problem

This approach is based on non-cooperative game theory:
It consists of a strategic game with two players where both players try to optimize, respectively, their utility function $f_1(x_1, x_2)$ et $f_2(x_1, x_2)$.
The two players do not communicate with each other and act in their own interest, the set of strategies containing all the pairs (x_1, x_2) representing each player's favorable strategy (i.e., which optimizes $f_i(x_1, x_2)$, $i = \{1, 2\}$) with a prediction of his opponent's preferred strategy. The solution of this game is called a Nash equilibrium (x_1^*, x_2^*).

We obtain a bi-matricial game formulated as a non-cooperative strategic problem $< I, X, F >$ where:

I: **The set of players** $I = \{f_1(x_1, x_2), f_2(x_1, x_2)\}$

X: **The set of strategies** $X_i=\{$extreme points of the bi-objective optimization problem.$\}$

F: **The set of payoffs** is calculated as follows:

$$\begin{cases} f_k(x_i, x_i) = & f_k(x_i, x_i) \\ f_k(x_i, x_j) = & \frac{f_k(X_i)+f_k(X_j)}{2} \end{cases} \quad k = \{1, 2\} \tag{1}$$

2.3 The proposed method

The goal of this method is to find all the Pareto optimal solutions for a bi-objective linear programming problem. The resolution is carried out in several steps as follows:

Data: A bi-objective linear programming problem.
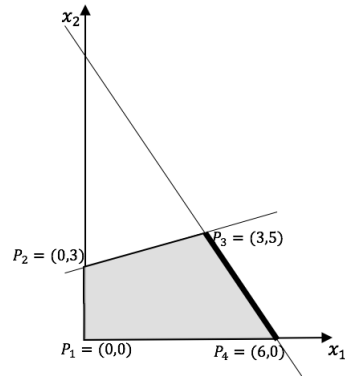**Result:** The Pareto optimal solutions of the problem.
initialization;
$S = \emptyset$ (Strategies);
Determine the decision set D;
Determine the extreme points using an enumeration method;
$S \leftarrow P_i$ (Extreme points of the problem);
Calculate the respective payoff of each player by the formula given by Equation 1;
Determine the Nash equlibrium $\{P_1, P_2\}$ of the game by the best answers method;
if *The obtained bases are adjacent* **then**
  | $\{P_1, P_2\}$ is the segment representing the efficient solutions
else

1. Find paths through the basic solutions of the problem, and compare the first points of each path.

2. Select the dominant point and return its path.

3. The returned path consists of efficient solutions.

end

Algorithm 1: Proposed algorithm.

3   Numerical example

Consider the following bi-objective linear program.

$$P_1 : \begin{cases} \textbf{\textit{Max}} & f_1 = 2x_1 + 3x_2 \\ \textbf{\textit{Max}} & f_2 = 4x_1 - x_2 \\ st & 5x_1 + 3x_2 \leq 30 \\ & -2x_1 + 3x_2 \leq 9 \\ & x_1 \geq 0, \ x_2 \geq 0 \end{cases}$$

(a) Illustration of the problem P_1

3.0.1   The associated game

$$J_1 : \begin{cases} I = \{f_1, f_2\} \\ X_k = \{(0,0), (0,3), (3,5), (6,0)\} \\ X = \prod_{k=1}^{2} X_k \end{cases}$$

Table 1 illustrates the payoff of f_1 and f_2:

The Nash equilibrium is given by (p_3, p_4). The segment joining the two points of the equilibrium represents the set of efficient solutions for problem P_1.

TAB. 1: Payoff table

		f_2			
		p_1	p_2	p_3	p_4
f_1	p_1	(0,0)	$\left(\frac{9}{2},-\frac{3}{2}\right)$	$\left(\frac{21}{2},\frac{7}{2}\right)$	(6,12)
	p_2	$\left(\frac{9}{2},-\frac{3}{2}\right)$	(9,-3)	(15,2)	$\left(\frac{21}{2},\frac{21}{2}\right)$
	p_3	$\left(\frac{21}{2},\frac{7}{2}\right)$	(15,2)	(21,7)	$\left(\frac{33}{2},\frac{31}{2}\right)$
	p_4	((6,12)	$\left(\frac{21}{2},\frac{21}{2}\right)$	$\left(\frac{32}{2},\frac{31}{2}\right)$	(12,24)

4 Conclusion

In this work, we have considered the problem of efficient data collection in wireless sensor networks. We have developed a multi-objective linear program to evaluate the compromise between the maximum amount of energy spent by sensor nodes and the length of the mobile sink route. We propose a new method for solving Bi-Objective Linear Programming problems using game theory. The presented method transforms a multi-objective linear optimization programming problem into a game theoretic one which can be solved using one of the Nash equilibrium search methods. This approach has been implemented to be tested and simulated on examples. The computational results confirm the effectiveness of the proposed method for small problems. We think that the results obtained in this work are very encouraging and deserve further study, and especially applications in the fields of IOT, such as optimization of information gathering in Wireless Sensor Networks, Data mining, etc.

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