



**HAL**  
open science

## A simple algorithm for stable order reduction of z-domain Laguerre models

Mihai Telescu, Nadia Iassamen, Pascale Cloastre, Noël Tanguy

► **To cite this version:**

Mihai Telescu, Nadia Iassamen, Pascale Cloastre, Noël Tanguy. A simple algorithm for stable order reduction of z-domain Laguerre models. *Signal Processing*, 2013, 13, pp.332-337. 10.1016/j.sigpro.2012.07.006 . hal-00749338

**HAL Id: hal-00749338**

**<https://hal.univ-brest.fr/hal-00749338>**

Submitted on 19 Feb 2014

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

**Title:** A simple algorithm for stable order reduction of z-domain Laguerre models

**Authors:** M. Telescu, N. Iassamen, P. Cloastre, N. Tanguy

**Authors affiliation :** Université Européenne de Bretagne, Université de Brest ; CNRS, UMR 6285, Lab-STICC, France

**Corresponding author:** Noël Tanguy  
Lab-STICC, UMR CNRS n°6285  
UFR Sciences et Techniques, 6, Av. Le Gorgeu  
Université de Bretagne Occidentale, C.S. 93837, 29238 BREST Cedex 3  
FRANCE  
E-mail: Noel.Tanguy@univ-brest.fr  
Tel.: (+33) 2.98.01.71.60, Fax: (33) 2.98.01.63.95

**Abstract:** Discrete-time Laguerre series are a well-known and efficient tool in system identification and modeling. This paper presents a simple solution for stable and accurate order reduction of systems described by a Laguerre model.

**Keywords:** discrete Laguerre functions; model order reduction; least-squares approximation

**Highlights:**

Discrete Laguerre series are useful tools to approximate signals or systems.

A fast “compression” of the Laguerre model to a low order rational fraction.

A very simple algorithm based on least squares optimization.

The reduced models are provably stable.

**Title:** A simple algorithm for stable order reduction of z-domain Laguerre models

**Authors:** M. Telescu, N. Iassamen, P. Cloastre, N. Tanguy

**Abstract:** Discrete-time Laguerre series are a well known and efficient tool in system identification and modeling. This paper presents a simple solution for stable and accurate order reduction of systems described by a Laguerre model.

**Keywords:** discrete Laguerre functions; model order reduction; least-squares approximation

## **1. Introduction**

Discrete Laguerre filters are simple and useful tools that have been successfully used in various fields of science. Several authors developed Laguerre-based methods for the identification and approximation of signals or systems [1-4], adaptive filtering [5-8] or filter design [9-11]. Specific applications include echo cancellation systems [12], delta demodulator circuits [13-14], equalizers [15], broadband signal generators [16], etc. Furthermore, in the field of nonlinear system identification, Laguerre basis functions can be used in the expansion of Wiener and Volterra kernels [17-18] with applications in physiological modeling [19], automatic control [20-21], nonlinear circuit modeling [22], etc.

Laguerre-based networks are classically seen as a compromise between FIR and IIR models [7,23-24]. On one hand they can model long impulse response systems more efficiently than FIR structures and, on the other hand, they offer a more straightforward design methodology with respect to a general IIR approach. Laguerre functions and filters only depend on a free parameter (a multiple-order single pole) that predefines the denominator of the resulting rational model. The choice of this parameter, which has been largely discussed in literature [25-32], is of great importance in a view to reduce the order of the Laguerre filters. Nevertheless, even when optimal procedures are used, Laguerre models can unfortunately fail to provide a desired accuracy vs. a given compactness and therefore classical or more general IIR filters must be used. In this paper we propose a method that allows deriving such low order rational fractions for systems described by Laguerre series. The technique is independent of the algorithm used to compute the Laguerre model and can be seen as a fast “compression” of the latter. The method boils down to a very simple algorithm based on least squares optimization leading to provably stable IIR filters.

The paper is organized as follows. Section II reminds the definition of the discrete-time Laguerre functions and some of the properties that may be used to efficiently compute the expansion coefficients. Section III directly addresses z-domain model order reduction and contains the core of this paper's theoretical contribution. Illustrative examples are provided in Section IV.

## **2. Laguerre-based modeling**

The discrete-time Laguerre functions  $\phi_n[k]$  are usually defined by their z-transform [1-2]

$$\Phi_n(z) \triangleq \sqrt{1-\lambda^2} \frac{z}{z-\lambda} \left( \frac{1-\lambda z}{z-\lambda} \right)^n, \quad n=0,1,2,\dots, \quad (1)$$

where  $\lambda$  is a real pole satisfying  $|\lambda| < 1$ . They form an orthonormal basis in  $\ell^2(\mathfrak{R}_+)$ . Consequently, any function  $f \in \ell^2(\mathfrak{R}_+)$  can be represented as a Laguerre series

$$f[k] = \sum_{n=0}^{\infty} c_n \phi_n[k], \quad (2)$$

where the Laguerre coefficients, also referred to as the Laguerre spectrum,  $\{c_n\}_{n \geq 0}$  are given by

$$c_n = \langle f, \phi_n \rangle = \sum_{k=0}^{\infty} f[k] \phi_n[k]. \quad (3)$$

In practice, the expansion is truncated at an order  $N$ ,

$$f_N[k] = \sum_{n=0}^{N-1} c_n \phi_n[k] \quad (4)$$

and the quadratic error  $\|f - f_N\|^2$  is minimal when the coefficients  $c_n$  are given by (3).

Let  $F(z)$  denote the z-transform of  $f[k]$ . Using (1) it can be written as

$$F(z) = \sum_{n=0}^{\infty} c_n \Phi_n(z) = \sqrt{1-\lambda^2} \frac{z}{z-\lambda} \sum_{n=0}^{\infty} c_n \left( \frac{1-\lambda z}{z-\lambda} \right)^n. \quad (5)$$

The bilinear transformation  $w = (z - \lambda)/(1 - \lambda z)$  which maps the open unit disk in z-domain onto itself can be used to derive the z-transform of the Laguerre spectrum, i.e.

$$G_0(w) \triangleq \sum_{n=0}^{\infty} c_n w^{-n} = \sqrt{1-\lambda^2} \frac{w}{w+\lambda} F\left(\frac{w+\lambda}{1+\lambda w}\right). \quad (6)$$

It is worth noting that transformation (6) preserves inner products that may be computed equivalently in time domain or in Laguerre spectral domain (w-domain) by

$$\langle f, f \rangle = \langle G_0, G_0 \rangle = \sum_{n=0}^{\infty} c_n^2. \quad (7)$$

Moreover if we set  $w = \exp(i\theta_j)$ , relation (6), in its truncated form, can be likened to a discrete Fourier transform and a FFT algorithm could then be used to compute the Laguerre coefficients from a frequency response [33]. To calculate the Laguerre spectrum from temporal data, the inner products (3) can be evaluated by a time-reverse method (see [34] or [25]). One should note, however, that depending on the available starting point (measured data in time domain, analytic impulse response, rational or irrational z-domain transfer function) several strategies may be used. An extended discussion of the latter is beyond the scope of the present paper which rather focuses on how to obtain a “compressed” representation once a Laguerre model is available.

### **3. Model order reduction**

The search for a rational approximation of  $F(z)$  given by (5) or equivalently of  $G_0(w)$  defined by (6), that is optimal in the sense of minimizing the quadratic error, is a nonlinear problem (with respect to the coefficients of the denominator). To circumvent this we propose a pencil-of-functions type method. In the search for accurate approximations a new quadratic error criterion, linear with respect to these parameters, is defined. One notes that the proposed method also has the advantage of preserving the first  $R$  coefficients of the Laguerre spectrum of the original function where  $R$  denotes the order of the reduced model. The same results may be achieved in continuous time using the derivations available in [35].

Let us define  $G_j(w)$  for  $j = 0, 1, \dots, R$  as follows

$$G_j(w) = \sum_{n=0}^{\infty} c_{n+j} w^{-n} = c_j + c_{j+1} w^{-1} + c_{j+2} w^{-2} + \dots \quad (8)$$

Note that, for  $j = 0$ , one can recognize equation (6). Consider now the following quantity

$$E(w) = \sum_{j=0}^R a_j G_j(w), \quad \text{with } a_R = 1, \quad (9)$$

whose energy we will seek to minimize. Clearly, this amounts to determine an approximation of  $G_R(w)$  using the other functions  $G_j(w)$ ,  $j=0,1,\dots,R-1$ . The optimal coefficients  $a_j$ ,  $j=0,1,\dots,R-1$ , in the sense of minimizing the quadratic error  $\|E\|^2$  are then obtained by solving a classical linear problem

$$\Psi \vec{a} = -\vec{b} \quad (10)$$

where  $\vec{a} = [a_0 \ a_1 \ \dots \ a_{R-1}]^T$ ,  $\Psi$  is a  $R \times R$  Gram matrix constituted of the inner products  $\psi_{i,j} = \langle G_i, G_j \rangle$  for  $i, j=0,1,\dots,R-1$ , and  $\vec{b} \triangleq [\psi_{0,R} \ \psi_{1,R} \ \dots \ \psi_{R-1,R}]^T$  ( $T$  denotes the transpose).

Note that the inner products  $\psi_{i,j} = \langle G_i, G_j \rangle$  only depend on the Laguerre spectrum of  $f[k]$  and are given by

$$\psi_{i,j} = \sum_{n=0}^{\infty} c_{n+i} c_{n+j} \quad (11)$$

Moreover, the following relation can be used to efficiently construct the Gram matrix

$$\psi_{i,j} = \psi_{i-1,j-1} - c_{i-1} c_{j-1} \quad \text{for } i, j=1,2,\dots,R. \quad (12)$$

Now, taking advantage of the following relation binding  $G_j(w)$  to  $G_0(w)$

$$G_j(w) = G_0(w) w^j - \sum_{n=0}^{j-1} c_n w^{j-n}, \quad (13)$$

and substituting (13) in (9) one can derive the following relation for  $G_0(w)$

$$G_0(w) = \frac{\sum_{j=1}^R a_j \sum_{n=0}^{j-1} c_n w^{j-n} + E(w)}{\sum_{j=0}^R a_j w^j}. \quad (14)$$

Notice that  $E(w)$  is the quantity defined in (9), and  $a_j$  ( $j=0,1,\dots,R-1$ ), solutions of (10), are the coefficients resulting from the minimization of  $\|E\|^2$ . Therefore  $\tilde{G}_0(w)$  defined by

$$\tilde{G}_0(w) = \frac{\sum_{j=1}^R a_j \sum_{n=0}^{j-1} c_n w^{j-n}}{\sum_{j=0}^R a_j w^j} \quad (15)$$

can be chosen as an approximation for  $G_0(w)$ . It's interesting to note that the approximation  $\tilde{G}_0(w)$  given in (15) leads to the conservation of the first  $R$  Laguerre coefficients of  $G_0(w)$  i.e.

$$\tilde{G}_0(w) = c_0 + c_1 w^{-1} + \dots + c_{R-1} w^{-(R-1)} + \tilde{c}_R w^{-R} + \tilde{c}_{R+1} w^{-(R+1)} + \dots$$

An approximation  $\tilde{F}(z)$  for  $F(z)$  is derived from (15) by the inverse transform  $z = (w + \lambda)/(1 + \lambda w)$  as follows

$$\tilde{F}(z) = \sqrt{1 - \lambda^2} \frac{z}{z - \lambda} \tilde{G}_0\left(\frac{z - \lambda}{1 - \lambda z}\right). \quad (16)$$

The algorithm for a reduced-order rational approximation of  $F(z)$  defined by its Laguerre spectrum is then very simple: first solve the linear system (10) and construct  $\tilde{G}_0(w)$  using (15), secondly apply the inverse transform (16) to obtain  $\tilde{F}(z)$ .

Note that the expansion of  $f[k]$  on the Laguerre basis is, in practice, truncated as in (4). One may use an energy criterion to choose the truncation order  $N$ . Above a certain order, the contribution of further coefficients in terms of energy usually becomes negligible. Selecting  $R$ , the order of the reduced model, is a more delicate issue and a recurring one in system theory. In the case of the proposed method the presence of a gap between the large and small singular values of the Gram matrix is an indication of manifest overfitting. More generally, the user may set a threshold on the singular values and choose  $R$  equal to the number of singular values above the threshold. A good practice is to keep  $R \ll N$ . As with all model order reduction methods the problem remains related to the application and boils down to an accuracy-versus-simplicity choice.

Remark:  $\tilde{F}(z)$  is asymptotically stable.

Proof: The Gram matrix  $\Psi$  is a real positive definite matrix. In the state-space representation, consider

the Lyapunov equation  $\Psi - A^T \Psi A = C$  where  $A \triangleq \left( \begin{array}{c|c} \tilde{0}^T & \\ \hline I & -\tilde{a} \end{array} \right)$  represents the companion form associated

with the denominator of  $\tilde{G}_0(w)$ . Taking into account that  $\tilde{a}$  satisfies (10) and using (12), the solution of

the Lyapunov equation is given by  $C = \tilde{c} \tilde{c}^T + M$  where  $\tilde{c} \triangleq [c_0 \ c_1 \ \dots \ c_{R-1}]^T$  and  $M \triangleq \left( \begin{array}{c|c} 0 & \tilde{0} \\ \hline \tilde{0}^T & \tilde{\varepsilon} \end{array} \right)$  with

$\varepsilon = \|E\|^2$ . As the quadratic error  $\varepsilon$  is always non-negative,  $C$  matrix is positive semi-definite. Therefore  $\tilde{G}_0(w)$  cannot have any pole outside the unit circle and thus  $\tilde{F}(z)$  cannot have any pole outside the unit disk. Furthermore, provided that  $\{A, \bar{c}\}$  is observable, the reduced model is asymptotically stable [36]. ■

#### **4. Examples**

To illustrate the method two examples are presented. First, we have considered the seventh-order transfer function of a supersonic jet engine inlet [37-38]

$$F(z) = z \frac{2.0434z^6 - 4.9825z^5 + 6.57z^4 - 5.8189z^3 + 3.636z^2 - 1.4105z + 0.2997}{z^7 - 2.46z^6 + 3.433z^5 - 3.333z^4 + 2.5460z^3 - 1.584z^2 + 0.7478z - 0.2520}. \quad (17)$$

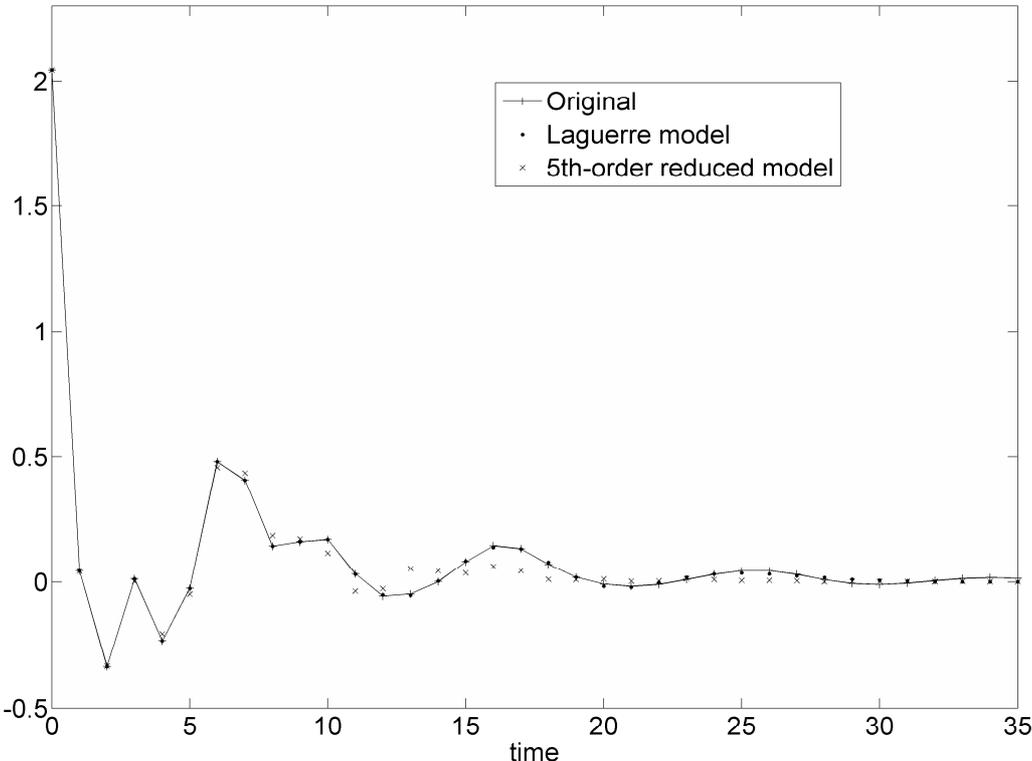
Using  $N = 20$  Laguerre functions defined for the parameter  $\lambda = 0.24$ , we have obtained a very good representation of the system (17) with a relative quadratic error of only  $4.83 \times 10^{-4}$ .

The value  $\lambda = 0.24$  for the Laguerre parameter has been obtained using the suboptimal method described in [27,30]. However this Laguerre model is of a relatively high order and a more compact model is desirable. Thus, using the presented method the following fifth-order reduced model has been derived for  $F(z)$

$$\tilde{F}(z) = z \frac{2.043z^4 - 3.057z^3 + 2.195z^2 - 1.545z + 0.8617}{z^5 - 1.518z^4 + 1.270z^3 - 1.032z^2 + 0.7539z - 0.3156}. \quad (18)$$

The impulse responses of the original transfer function, of the Laguerre model and of the reduced-order model are shown in Fig.1. The relative quadratic error  $\|f - \tilde{f}\|^2 / \|f\|^2$  of this fifth-order reduced model is  $1.06 \times 10^{-2}$ . Table 1 compares the quality of the impulse response of the proposed model (LG5) with those of models derived through Balanced Realization (BR5, see [39]), Weighted Impulse Response Gramian (WIRG5 see [36]) and Least-Squares with Scaling (LSS5, see [37]), Generalized Impulse Response Gramian (GIRG5 see [38]) and Laguerre-SVD (LSVD5 see [40]). The latter was implemented using the same Laguerre parameter as LG5 ( $\lambda = 0.24$ ), and shares the property of matching the first  $R$  Laguerre coefficients of the original system. Table 1 shows the good quality of the proposed model-order reduction procedure. It should be noted that the method does not necessarily preserve the static gain of  $F(z)$  and model (19) presents an 8.95% error on it. However, a correction of the supplied numerator is always possible to preserve DC gain without dramatically deteriorating the overall model.

The results in Table 1 confirm the generally accepted conclusion that gramian-based methods (as BR5, WIRG5, GIRG5) are accurate. The price to pay for this accuracy is usually related to computational resources required to evaluate gramians. For the technique presented in this paper, relation (12) partially compensates this disadvantage. Projection based subspace Krylov methods (as LSVD5) are known to be fast, however they may lead to comparatively larger quadratic errors. Most of these methods yields reduced order model preserving specific properties of the original system: time moments, Markov parameters or Laguerre coefficients. A method proposing the conservation of a mix of time moments, Markov parameter and power moments was proposed in [41-42].



**Fig. 1:** Impulse responses of the original system and of its Laguerre model and of its 5th-order reduced model

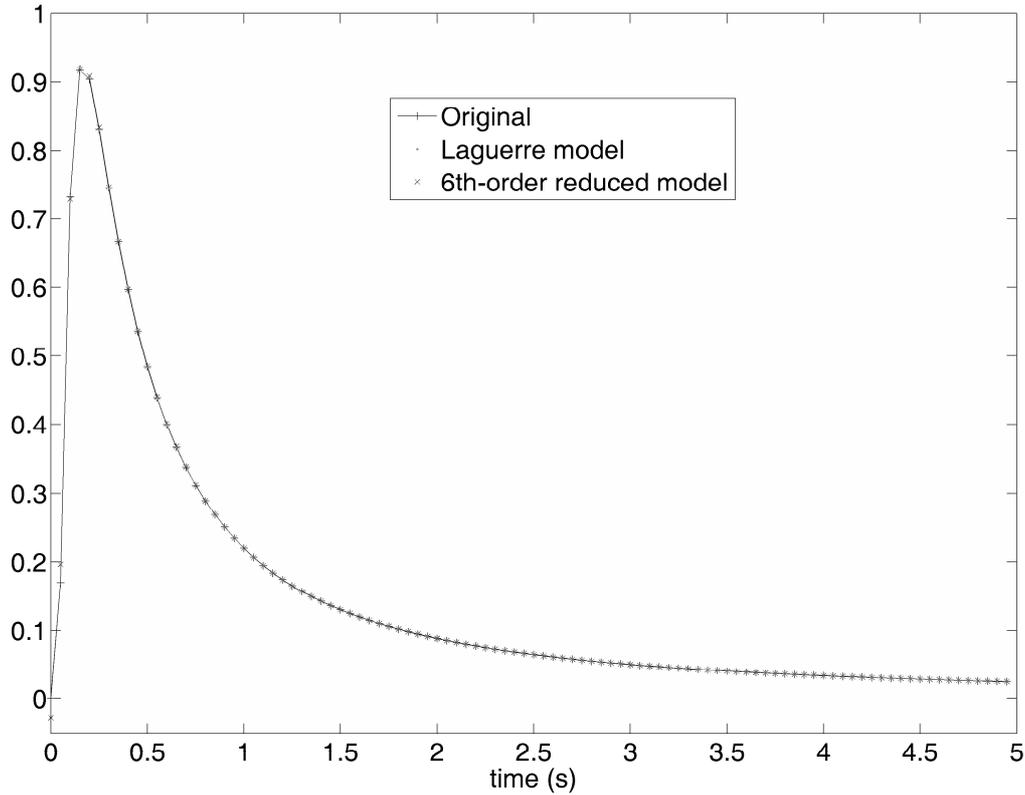
Models	Impulse Error
<b>LG5</b>	$1.06 \times 10^{-2}$
GIRG5	$1.27 \times 10^{-2}$
BR5	$1.19 \times 10^{-2}$
WIRG5	$1.13 \times 10^{-2}$
LSS5	$2.36 \times 10^{-2}$
LSVD5	$7.43 \times 10^{-2}$

**Table 1:** Quality comparison of different methods

The second example deals with the compact rational modeling of an infinite-dimensional system. The considered system is an underwater cable, whose irrational transfer function is [43-44]

$$\hat{f}(s) = e^{-\sqrt{K}s} \text{ (with } K = 1). \quad (19)$$

The impulse response of this system is given in Fig. 2.



**Fig. 2:** Impulse responses of the original system, of its 32th-order Laguerre model and of its 6th-order reduced model

Four different discrete-time Laguerre approximations of this system, with orders ranging from 8 to 32 have been computed using a sampling period of  $T_s = 0.05s$ . The Laguerre spectra have been computed using the FFT-based algorithm described in [33]. The choice  $\lambda = 0.88$  for the Laguerre parameter has been obtained using the method described in [27,30]. These four Laguerre models have subsequently been used to compute four sixth-order rational models (LG6) according to the technique described in Section 3. The relative quadratic errors computed over the first 1000 samples of the model impulse responses are reported in Table 2. This table shows that the quality of the reduced model is very dependant on the quality of the Laguerre model used for its construction. Moreover, Table 2 shows that important order reduction can be achieved using this technique. Indeed, the last model almost perfectly

mimics the response of the Laguerre network it was derived from while achieving a complexity reduction from order 32 down to order 6. The impulse responses of these last models are compared in Fig. 2. This example also shows that the method is useful in modeling applications dealing with irrational transfer functions.

Laguerre model order	Laguerre model error	LG6 model error
8	$1.43 \times 10^{-2}$	$1.43 \times 10^{-2}$
16	$4.12 \times 10^{-3}$	$4.69 \times 10^{-3}$
24	$6.02 \times 10^{-4}$	$6.50 \times 10^{-4}$
32	$2.39 \times 10^{-4}$	$2.43 \times 10^{-4}$

**Table 2:** Relative quadratic error computed over the interval 0-5s

## **5. Conclusion**

A simple algorithm has been proposed to provide reduced-order rational models of systems described by discrete Laguerre functions. The method is based on the minimization of a quadratic error that requires inverting a matrix of size  $R \times R$  where  $R$  is the order of the desired compact model. Models provided by this method are provably stable and conserve the  $R$ -first Laguerre coefficients of the original system. An example illustrates the good results provided by the presented technique.

## **Acknowledgment**

The authors wish to acknowledge Brittany Region for its financial support.

## **References**

- [1] C.R. Arnold, Laguerre functions and the Laguerre network. Their properties and digital simulation, Massachusetts Institute of Technology Lincoln Laboratory, Technical Note 1966-28 (May 1966).
- [2] R.E. King, P.N. Paraskevopoulos, Digital Laguerre filters, Circuit theory and applications 5 (1977) 81-91.
- [3] U. Nurges, Laguerre models in problems of approximation and identification of discrete systems, Automat. Remote Control 48 (1987) 346-352.
- [4] P.S.C. Heuberger, P.M.J. Van den Hof, B. Wahlberg, Modeling and Identification with Rational Orthonormal Basis Functions, Springer-Verlag (2005).
- [5] A.C. Den Brinker, Adaptive modified Laguerre filters, Signal Processing 31 (1) (1993) 69-79.
- [6] A.C. Den Brinker, Laguerre-domain adaptive filters, IEEE Trans. on Signal Processing 42 (4) (1994) 953-956.

- [7] Z. Fejzo, H. Lev-Ari, Adaptive Laguerre lattice filters, *IEEE Trans. on Signal Processing* 45 (12) (1997) 3006-3016.
- [8] M.A. Masnadi-Shirazi, M. Ghasemi, Adaptive Laguerre network realization, *Signal Processing* 80 (2000) 2169-2186.
- [9] M.A. Masnadi-Shirazi, M. Aleshams, Laguerre discrete-time filter design, *Computers & Electrical Engineering* 29 (1) (2003) 173–192.
- [10] H.H. Dam, A. Cantoni, S. Nordholm, K.L. Teo, Digital Laguerre Filter Design With Maximum Passband-to-Stopband Energy Ratio Subject to Peak and Group Delay Constraints *IEEE Trans. on Circuits and Systems I : Regular Papers* 53 (5) (2006) 1108-1118.
- [11] M.A. Masnadi-Shirazi, A. Zollanvari, Complex digital Laguerre filter design with weighted least square error subject to magnitude and phase constraints, *Signal Processing* 88 (4) (2008) 796-810.
- [12] G.W. Davidson, D.D. Falconer, Reduced complexity echo cancellation using orthonormal functions, *IEEE Trans. on Circuits and Systems* 38 (1) (1991) 20-28.
- [13] S.S. Abeysekera, Z. Zang, Optimal Laguerre filters for sigma-delta demodulator circuits, *Signal Processing* 80 (1) (2000) 205-209.
- [14] S.S. Abeysekera, Recursive Laguerre and Kalman filters as efficient full-rate sigma–delta ( $\Sigma$ - $\Delta$ ) demodulators, *Signal Processing* 87 (3) (2007) 417-431.
- [15] S.S. Abeysekera, Y. Ye, Zero-based equalizers for single-input single-output and single-input multiple-output channels, *Signal Processing* 88 (7) (2008) 1868-1880.
- [16] S.R. Seydnejad, R. Ebrahimi, Broadband beamforming using Laguerre filters, *Signal Processing* 92 (4) (2012) 1093-1100.
- [17] J. Amorocho, A. Brandstetter, Determination of nonlinear functional response functions in rainfall runoff processes, *Water Resources Research* 7 (5) (1971) 1087-1101.
- [18] A. Watanabe, L. Stark, Kernel method for nonlinear analysis: identification of a biological control system, *Mathematical Biosciences* 27 (1975) 99-108.
- [19] V.Z. Marmarelis, Identification of Nonlinear Biological Systems Using Laguerre Expansions of Kernels, *Annals of Biomedical Engineering* 21 (2) (1993) 573-589.
- [20] G. Dumont, Y. Fu, Non-linear adaptive control via Laguerre expansion of Volterra kernels, *Int. J. Adapt. Control Signal Process.* 7 (1993) 367-382.
- [21] R.J.G.B. Campello, G. Favier, W.C. Amaral, Optimal expansions of discrete-time Volterra models using Laguerre functions, *Automatica* 40 (2004) 815-822.

- [22] M.G. Telescu, I.S. Stievano, F.G. Canavero, N. Tanguy, An Application of Volterra Series to IC Buffer Models, 14th IEEE Workshop on Signal Propagation On Interconnects, Hildesheim, Germany, May 9-12 2010, pp. 93-96.
- [23] M.A. Masnadi-Shirazi, M. Aleshams, Laguerre discrete-time filter design, *Computers and Electrical Engineering* 29 (2003) 173-192.
- [24] A. Zollanvari, M.A. Masnadi-Shirazi, A class of comprehensive constraints for design of PCWLSE Laguerre and FIR filters : A boost in performance, *Signal Processing* 90 (4) (2010) 1118-1130.
- [25] M.A. Masnadi-Shirazi, Optimum synthesis of linear discrete-time systems using orthogonal Laguerre sequences, PhD, University of New Mexico, Albuquerque, New Mexico (1990).
- [26] M.A. Masnadi-Shirazi, N. Ahmed, Optimum Laguerre networks for a class of discrete-time systems, *IEEE Trans. Signal Process.* 39 (9) (1991) 2104-2108.
- [27] Y. Fu, G.A. Dumont, An optimum time scale for discrete Laguerre network, *IEEE Trans. on Automatic Control.* 38 (6) (1993) 934-938.
- [28] T. Oliveira e Silva, Optimality conditions for truncated Laguerre network, *IEEE Trans. on Signal Processing* 42 (1994) 2528-2530.
- [29] T. Oliveira e Silva, On the determination of the optimal pole position of Laguerre filters. *IEEE Trans. on Signal Processing* 43 (9) (1995) 2079-2087.
- [30] N. Tanguy, P. Vilbé, L.C. Calvez, Optimum choice of free parameter in orthonormal approximations, *IEEE Trans. Automatic Control* 40 (10) (1995) 1811-1813.
- [31] N. Tanguy, R. Morvan, P. Vilbé, L.C. Calvez, Online optimization of the time scale in adaptive Laguerre-based filters, *IEEE Trans. on Signal Processing* 48 (4) (2000) 1184-1187.
- [32] A.C. den Brinker, B.E. Sarroukh, Pole optimisation in adaptive Laguerre filtering, *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP '04)*, 17-21 May 2004, pp. 37-40.
- [33] J. Bokor, F. Schipp, Approximate Identification in Laguerre and Kautz Bases, *Automatica* 34 (4) (1998) 463-468.
- [34] M. Roesch, Identification des systèmes linéaires par intégration et récurrence, Thèse, Université de Nancy I, Nancy (1973).
- [35] A. Amghayrir, N. Tanguy, P. Bréhonnet, P. Vilbé, L.C. Calvez, Laguerre-Gram reduced-order modeling, *IEEE Trans. on Automatic Control* 50 (9) (2005) 1432-1435.
- [36] V. Sreeram, P. Athoklis, Model reduction of linear discrete systems via weighted impulse response Gramians, *International Journal of Control* 53 (1) (1991) 129-144.

- [37] R.J. Lalonde, T.T. Hartley, J.A. De Abreu-Garcia, Least-squares model order reduction enhancements, *IEEE Trans. Indus. Electronics* 40 (6) (1993) 533-541.
- [38] S. Azou, P. Bréhonnet, P. Vilbé, L.C. Calvez, A New Discrete Impulse Response Gramian and its Application to Model Reduction, *IEEE Trans. on Automatic Control* 45 (3) (2000) 533-537.
- [39] L. Pernebo, L.M. Silverman, Model reduction via balanced state space representation, *IEEE Trans. Automatic Control* 27 (1982) 382–387.
- [40] L. Knockaert, D. De Zutter, Stable Laguerre-SVD reduced-order modeling, *IEEE Trans. Circuits Syst. I* 50 (4) (2003) 576-579.
- [41] V. Sreeram, P. Agathoklis, The Discrete-time  $q$ -Markov Covers Models with Improved Low-Frequency Approximation, *IEEE Trans. on Automatic Control* 39 (1994) 1102-1105.
- [42] V. Sreeram, On the Generalized  $q$ -Markov Cover Models for Discrete-Time Systems, *IEEE Trans. on Automatic Control* 39 (1994) 2502-2505.
- [43] C. Hsu, and D. Hou, Linear approximation of fractional transfer functions of distributed parameter systems, *Electronics Letters* 26 (15) (1990) 1211-1213.
- [44] N. Tanguy, P. Bréhonnet, P. Vilbé, L.C. Calvez, Gram matrix of a Laguerre model: application to model reduction of irrational transfer function, *Signal Processing* 85 (3) (2005) 651-655.