A simple algorithm for stable order reduction of z-domain Laguerre models

Mihai Telescu, Nadia Iassamen, Pascale Cloastre, Noël Tanguy

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Authors: M. Telescu, N. Iassamen, P. Cloastre, N. Tanguy

Authors affiliation : Université Européenne de Bretagne, Université de Brest ; CNRS, UMR 6285, Lab-STICC, France

Corresponding author: Noël Tanguy
Lab-STICC, UMR CNRS n°6285
UFR Sciences et Techniques, 6, Av. Le Gorgeu
Université de Bretagne Occidentale, C.S. 93837, 29238 BREST Cedex 3
FRANCE
E-mail: Noel.Tanguy@univ-brest.fr
Tel.: (+33) 2.98.01.71.60, Fax: (33) 2.98.01.63.95

Abstract: Discrete-time Laguerre series are a well-known and efficient tool in system identification and modeling. This paper presents a simple solution for stable and accurate order reduction of systems described by a Laguerre model.

Keywords: discrete Laguerre functions; model order reduction; least-squares approximation

Highlights:

Discrete Laguerre series are useful tools to approximate signals or systems.

A fast “compression” of the Laguerre model to a low order rational fraction.

A very simple algorithm based on least squares optimization.

The reduced models are provably stable.

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1. Introduction

Discrete Laguerre filters are simple and useful tools that have been successfully used in various fields of science. Several authors developed Laguerre-based methods for the identification and approximation of signals or systems [1-4], adaptive filtering [5-8] or filter design [9-11]. Specific applications include echo cancellation systems [12], delta demodulator circuits [13-14], equalizers [15], broadband signal generators [16], etc. Furthermore, in the field of nonlinear system identification, Laguerre basis functions can be used in the expansion of Wiener and Volterra kernels [17-18] with applications in physiological modeling [19], automatic control [20-21], nonlinear circuit modeling [22], etc.

Laguerre-based networks are classically seen as a compromise between FIR and IIR models [7,23-24]. On one hand they can model long impulse response systems more efficiently than FIR structures and, on the other hand, they offer a more straightforward design methodology with respect to a general IIR approach. Laguerre functions and filters only depend on a free parameter (a multiple-order single pole) that predefines the denominator of the resulting rational model. The choice of this parameter, which has been largely discussed in literature [25-32], is of great importance in a view to reduce the order of the Laguerre filters. Nevertheless, even when optimal procedures are used, Laguerre models can unfortunately fail to provide a desired accuracy vs. a given compactness and therefore classical or more general IIR filters must be used. In this paper we propose a method that allows deriving such low order rational fractions for systems described by Laguerre series. The technique is independent of the algorithm used to compute the Laguerre model and can be seen as a fast “compression” of the latter. The method boils down to a very simple algorithm based on least squares optimization leading to provably stable IIR filters.

The paper is organized as follows. Section II reminds the definition of the discrete-time Laguerre functions and some of the properties that may be used to efficiently compute the expansion coefficients. Section III directly addresses z-domain model order reduction and contains the core of this paper's theoretical contribution. Illustrative examples are provided in Section IV.

2. Laguerre-based modeling

The discrete-time Laguerre functions $\phi_n[k]$ are usually defined by their z-transform [1-2]

$$\Phi_n(z) \mathrel{\hat{=}} \sqrt{1-\lambda^2}\,\frac{z}{z-\lambda}\left(\frac{1-\lambda z}{z-\lambda}\right)^n, \quad n = 0,1,2\ldots, \qquad (1)$$

where λ is a real pole satisfying $|\lambda|<1$. They form an orthonormal basis in $\ell^2(\Re_+)$. Consequently, any function $f \in \ell^2(\Re_+)$ can be represented as a Laguerre series

$$f[k] = \sum_{n=0}^{\infty} c_n \phi_n[k], \qquad (2)$$

where the Laguerre coefficients, also referred to as the Laguerre spectrum, $\{c_n\}_{n\geq 0}$ are given by

$$c_n = \langle f, \phi_n \rangle = \sum_{k=0}^{\infty} f[k]\phi_n[k]. \qquad (3)$$

In practice, the expansion is truncated at an order *N*,

$$f_N[k] = \sum_{n=0}^{N-1} c_n \phi_n[k] \qquad (4)$$

and the quadratic error $\|f - f_N\|^2$ is minimal when the coefficients c_n are given by (3).

Let $F(z)$ denote the z-transform of $f[k]$. Using (1) it can be written as

$$F(z) = \sum_{n=0}^{\infty} c_n \Phi_n(z) = \sqrt{1-\lambda^2}\,\frac{z}{z-\lambda}\sum_{n=0}^{\infty} c_n \left(\frac{1-\lambda z}{z-\lambda}\right)^n. \qquad (5)$$

The bilinear transformation $w = (z-\lambda)/(1-\lambda z)$ which maps the open unit disk in *z*-domain onto itself can be used to derive the z-transform of the Laguerre spectrum, i.e.

$$G_0(w) \mathrel{\hat{=}} \sum_{n=0}^{\infty} c_n w^{-n} = \sqrt{1-\lambda^2}\,\frac{w}{w+\lambda}\,F\!\left(\frac{w+\lambda}{1+\lambda w}\right). \qquad (6)$$

It is worth noting that transformation (6) preserves inner products that may be computed equivalently in time domain or in Laguerre spectral domain (w-domain) by

$$\langle f, f \rangle = \langle G_0, G_0 \rangle = \sum_{n=0}^{\infty} c_n^2 . \qquad (7)$$

Moreover if we set $w = \exp(i\theta_j)$, relation (6), in its truncated form, can be likened to a discrete Fourier transform and a FFT algorithm could then be used to compute the Laguerre coefficients from a frequency response [33]. To calculate the Laguerre spectrum from temporal data, the inner products (3) can be evaluated by a time-reverse method (see [34] or [25]). One should note, however, that depending on the available starting point (measured data in time domain, analytic impulse response, rational or irrational z-domain transfer function) several strategies may be used. An extended discussion of the latter is beyond the scope of the present paper which rather focuses on how to obtain a "compressed" representation once a Laguerre model is available.

3. Model order reduction

The search for a rational approximation of $F(z)$ given by (5) or equivalently of $G_0(w)$ defined by (6), that is optimal in the sense of minimizing the quadratic error, is a nonlinear problem (with respect to the coefficients of the denominator). To circumvent this we propose a pencil-of-functions type method. In the search for accurate approximations a new quadratic error criterion, linear with respect to these parameters, is defined. One notes that the proposed method also has the advantage of preserving the first R coefficients of the Laguerre spectrum of the original function where R denotes the order of the reduced model. The same results may be achieved in continuous time using the derivations available in [35].

Let us define $G_j(w)$ for $j = 0,1,..., R$ as follows

$$G_j(w) = \sum_{n=0}^{\infty} c_{n+j} w^{-n} = c_j + c_{j+1} w^{-1} + c_{j+2} w^{-2} + ... \quad (8)$$

Note that, for $j = 0$, one can recognize equation (6). Consider now the following quantity

$$\mathrm{E}(w) = \sum_{j=0}^{R} a_j G_j(w), \quad \text{with } a_R = 1, \quad (9)$$

whose energy we will seek to minimize. Clearly, this amounts to determine an approximation of $G_R(w)$ using the other functions $G_j(w)$, $j=0,1,...,R-1$. The optimal coefficients a_j, $j=0,1,...,R-1$, in the sense of minimizing the quadratic error $\|E\|^2$ are then obtained by solving a classical linear problem

$$\Psi\vec{a} = -\vec{b} \qquad (10)$$

where $\vec{a} = [a_0\ a_1 \dots a_{R-1}]^T$, Ψ is a $R\times R$ Gram matrix constituted of the inner products $\psi_{i,j} = \langle G_i, G_j\rangle$ for $i, j = 0,1,\cdots, R-1$, and $\vec{b} \mathrel{\hat{=}} [\psi_{0,R}\ \psi_{1,R} \cdots \psi_{R-1,R}]^T$ (^{T} denotes the transpose).

Note that the inner products $\psi_{i,j} = \langle G_i, G_j\rangle$ only depend on the Laguerre spectrum of $f[k]$ and are given by

$$\psi_{i,j} = \sum_{n=0}^{\infty} c_{n+i}c_{n+j}\ . \qquad (11)$$

Moreover, the following relation can be used to efficiently construct the Gram matrix

$$\psi_{i,j} = \psi_{i-1,j-1} - c_{i-1}c_{j-1} \qquad \text{for } i, j = 1,2,\cdots, R\ . \qquad (12)$$

Now, taking advantage of the following relation binding $G_j(w)$ to $G_0(w)$

$$G_j(w) = G_0(w)w^j - \sum_{n=0}^{j-1} c_n w^{j-n}\ , \qquad (13)$$

and substituting (13) in (9) one can derive the following relation for $G_0(w)$

$$G_0(w) = \frac{\sum_{j=1}^{R} a_j \sum_{n=0}^{j-1} c_n w^{j-n} + E(w)}{\sum_{j=0}^{R} a_j w^j}\ . \qquad (14)$$

Notice that $E(w)$ is the quantity defined in (9), and a_j ($j=0,1,...,R-1$), solutions of (10), are the coefficients resulting from the minimization of $\|E\|^2$. Therefore $\tilde{G}_0(w)$ defined by

$$\tilde{G}_0(w) = \frac{\sum_{j=1}^{R} a_j \sum_{n=0}^{j-1} c_n w^{j-n}}{\sum_{j=0}^{R} a_j w^j} \qquad (15)$$

can be chosen as an approximation for $G_0(w)$. It's interesting to note that the approximation $\tilde{G}_0(w)$ given in (15) leads to the conservation of the first R Laguerre coefficients of $G_0(w)$ i.e.

$$\tilde{G}_0(w) = c_0 + c_1 w^{-1} + \ldots + c_{R-1} w^{-(R-1)} + \tilde{c}_R w^{-R} + \tilde{c}_{R+1} w^{-(R+1)} + \ldots$$

An approximation $\tilde{F}(z)$ for $F(z)$ is derived from (15) by the inverse transform $z = (w+\lambda)/(1+\lambda w)$ as follows

$$\tilde{F}(z) = \sqrt{1-\lambda^2}\,\frac{z}{z-\lambda}\,\tilde{G}_0\left(\frac{z-\lambda}{1-\lambda z}\right). \qquad (16)$$

The algorithm for a reduced-order rational approximation of $F(z)$ defined by its Laguerre spectrum is then very simple: first solve the linear system (10) and construct $\tilde{G}_0(w)$ using (15), secondly apply the inverse transform (16) to obtain $\tilde{F}(z)$.

Note that the expansion of $f[k]$ on the Laguerre basis is, in practice, truncated as in (4). One may use an energy criterion to choose the truncation order N. Above a certain order, the contribution of further coefficients in terms of energy usually becomes negligible. Selecting R, the order of the reduced model, is a more delicate issue and a recurring one in system theory. In the case of the proposed method the presence of a gap between the large and small singular values of the Gram matrix is an indication of manifest overfitting. More generally, the user may set a threshold on the singular values and choose R equal to the number of singular values above the threshold. A good practice is to keep $R << N$. As with all model order reduction methods the problem remains related to the application and boils down to an accuracy-versus-simplicity choice.

Remark: $\tilde{F}(z)$ is asymptotically stable.

Proof: The Gram matrix Ψ is a real positive definite matrix. In the state-space representation, consider the Lyapunov equation $\Psi - A^T \Psi A = C$ where $A \triangleq \left(\begin{array}{c|c} \vec{0}^T & -\vec{a} \\ I & \end{array}\right)$ represents the companion form associated with the denominator of $\tilde{G}_0(w)$. Taking into account that $\vec{a}$ satisfies (10) and using (12), the solution of the Lyapunov equation is given by $C = \vec{c}\vec{c}^{\,T} + M$ where $\vec{c} \triangleq [c_0\ c_1 \ldots c_{R-1}]^T$ and $M \triangleq \left(\begin{array}{c|c} 0 & \vec{0} \\ \hline \vec{0}^T & \varepsilon \end{array}\right)$ with

$\varepsilon = \|E\|^2$. As the quadratic error ε is always non-negative, C matrix is positive semi-definite. Therefore $\tilde{G}_0(w)$ cannot have any pole outside the unit circle and thus $\tilde{F}(z)$ cannot have any pole outside the unit disk. Furthermore, provided that $\{A, \vec{c}\}$ is observable, the reduced model is asymptotically stable [36]. ■

4. Examples

To illustrate the method two examples are presented. First, we have considered the seventh-order transfer function of a supersonic jet engine inlet [37-38]

$$F(z) = z\frac{2.0434z^6 - 4.9825z^5 + 6.57z^4 - 5.8189z^3 + 3.636z^2 - 1.4105z + 0.2997}{z^7 - 2.46z^6 + 3.433z^5 - 3.333z^4 + 2.5460z^3 - 1.584z^2 + 0.7478z - 0.2520}. \quad (17)$$

Using $N = 20$ Laguerre functions defined for the parameter $\lambda = 0.24$, we have obtained a very good representation of the system (17) with a relative quadratic error of only 4.83×10^{-4}.

The value $\lambda = 0.24$ for the Laguerre parameter has been obtained using the suboptimal method described in [27,30]. However this Laguerre model is of a relatively high order and a more compact model is desirable. Thus, using the presented method the following fifth-order reduced model has been derived for $F(z)$

$$\tilde{F}(z) = z\frac{2.043z^4 - 3.057z^3 + 2.195z^2 - 1.545z + 0.8617}{z^5 - 1.518z^4 + 1.270z^3 - 1.032z^2 + 0.7539z - 0.3156}. \quad (18)$$

The impulse responses of the original transfer function, of the Laguerre model and of the reduced-order model are shown in Fig.1. The relative quadratic error $\left\|f - \tilde{f}\right\|^2 \Big/ \|f\|^2$ of this fifth-order reduced model is 1.06×10^{-2}. Table 1 compares the quality of the impulse response of the proposed model (LG5) with those of models derived through Balanced Realization (BR5, see [39]), Weighted Impulse Response Gramian (WIRG5 see [36]) and Least-Squares with Scaling (LSS5, see [37]), Generalized Impulse Response Gramian (GIRG5 see [38]) and Laguerre-SVD (LSVD5 see [40]). The latter was implemented using the same Laguerre parameter as LG5 ($\lambda = 0.24$), and shares the property of matching the first R Laguerre coefficients of the original system. Table 1 shows the good quality of the proposed model-order reduction procedure. It should be noted that the method does not necessarily preserve the static gain of $F(z)$ and model (19) presents an 8.95% error on it. However, a correction of the supplied numerator is always possible to preserve DC gain without dramatically deteriorating the overall model.

The results in Table 1 confirm the generally accepted conclusion that gramian-based methods (as BR5, WIRG5, GIRG5) are accurate. The price to pay for this accuracy is usually related to computational resources required to evaluate gramians. For the technique presented in this paper, relation (12) partially compensates this disadvantage. Projection based subspace Krylov methods (as LSVD5) are known to be fast, however they may lead to comparatively larger quadratic errors. Most of these methods yields reduced order model preserving specific properties of the original system: time moments, Markov parameters or Laguerre coefficients. A method proposing the conservation of a mix of time moments, Markov parameter and power moments was proposed in [41-42].

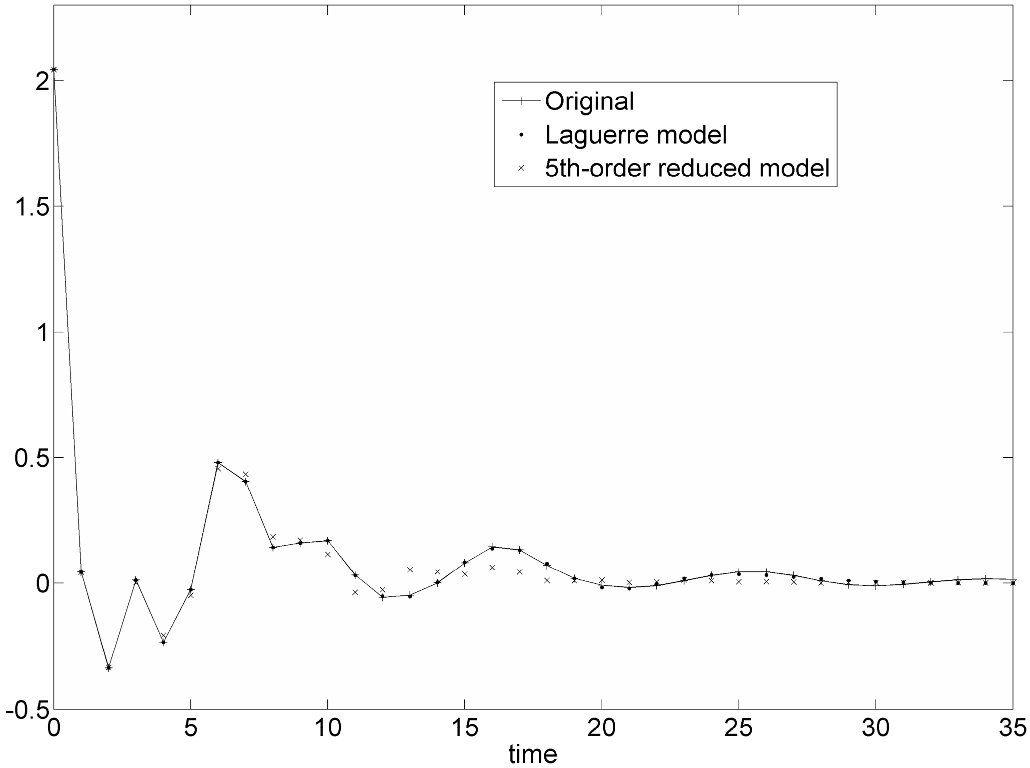

Fig. 1: Impulse responses of the original system and of its Laguerre model and of its 5th-order reduced model

Models	Impulse Error
LG5	1.06×10^{-2}
GIRG5	1.27×10^{-2}
BR5	1.19×10^{-2}
WIRG5	1.13×10^{-2}
LSS5	2.36×10^{-2}
LSVD5	7.43×10^{-2}

Table 1: Quality comparison of different methods

The second example deals with the compact rational modeling of an infinite-dimensional system. The considered system is an underwater cable, whose irrational transfer function is [43-44]

$$\hat{f}(s) = e^{-\sqrt{Ks}} \text{ (with } K = 1). \quad (19)$$

The impulse response of this system is given in Fig. 2.

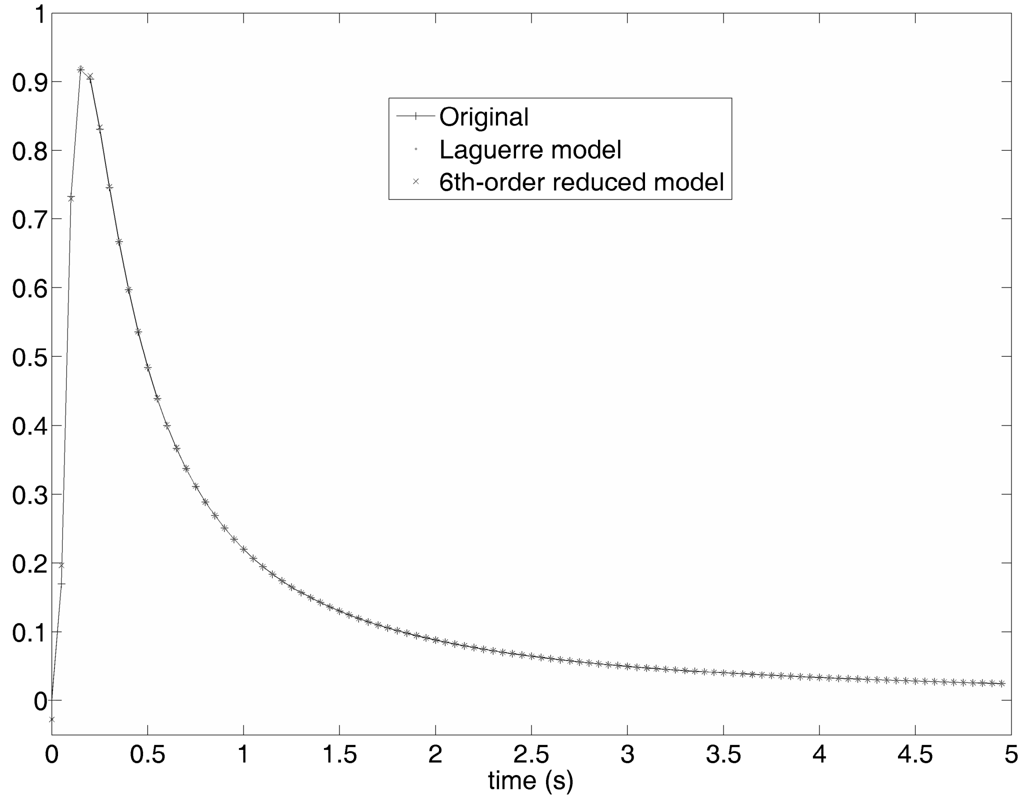

Fig. 2: Impulse responses of the original system, of its 32th-order Laguerre model and of its 6th-order reduced model

Four different discrete-time Laguerre approximations of this system, with orders ranging from 8 to 32 have been computed using a sampling period of Ts= 0.05s. The Laguerre spectra have been computed using the FFT-based algorithm described in [33]. The choice $\lambda = 0.88$ for the Laguerre parameter has been obtained using the method described in [27,30]. These four Laguerre models have subsequently been used to compute four sixth-order rational models (LG6) according to the technique described in Section 3. The relative quadratic errors computed over the first 1000 samples of the model impulse responses are reported in Table 2. This table shows that the quality of the reduced model is very dependant on the quality of the Laguerre model used for its construction. Moreover, Table 2 shows that important order reduction can be achieved using this technique. Indeed, the last model almost perfectly mimics the response of the Laguerre network it was derived from while achieving a complexity reduction from order 32 down to order 6. The impulse responses of these last models are compared in Fig. 2. This example also shows that the method is useful in modeling applications dealing with irrational transfer functions.

Laguerre model order	Laguerre model error	**LG6 model error**
8	1.43×10^{-2}	1.43×10^{-2}
16	4.12×10^{-3}	4.69×10^{-3}
24	6.02×10^{-4}	6.50×10^{-4}
32	2.39×10^{-4}	2.43×10^{-4}

Table 2: Relative quadratic error computed over the interval 0-5s

5. Conclusion

A simple algorithm has been proposed to provide reduced-order rational models of systems described by discrete Laguerre functions. The method is based on the minimization of a quadratic error that requires inverting a matrix of size $R\times R$ where R is the order of the desired compact model. Models provided by this method are provably stable and conserve the R-first Laguerre coefficients of the original system. An example illustrates the good results provided by the presented technique.

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