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# Blind CFO estimation for OFDM-IDMA system in Rayleigh fading multipath channel

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**Abstract:** We address the problem of carrier frequency offset (CFO) in Orthogonal Frequency Division Multiplexing (OFDM) communications systems in the context of Interleaved Division Multiple Access (IDMA). We propose a technique to adapt a method of blind CFO estimation, called CFO estimation-Syndrome Function Minimization (C-SFM), to the context of a system combining IDMA and OFDM where a quasi-static Rayleigh fading multipath channel is considered. Performance in terms of Mean Squared Error (MSE) of carrier frequency offset is studied for a different number of users. In order to evaluate the robustness of the C-SFM method, we compare the performance of OFDM-IDMA system using C-SFM technique with that of a perfect synchronous system. Our results show that, for high  $E_b/N_0$ , the two systems have the same performance.

**Key words:** Blind carrier frequency offset estimation, OFDM, quasi-static Rayleigh fading multipath channel, IDMA.

## INTRODUCTION

In decentralized networks such as ad-hoc networks, one of the major challenges concerns the synchronization in the context of multi-user communications. The synchronization techniques based on pilot symbols decrease signaling for higher spectral efficiency. So, they are not suited for user detection in multi-user system. The authors of [HSD06] proposed a new effective technique for blind frame synchronization in the multi-user type IDMA. This latter is able to cancel the Multiple Access Interference (MAI) using the interleaving and the iterative multiuser detection [LPL03, MLJ06b]. Unlike of CDMA, it uses a same spreading code for each user. The potential of blind frame synchronization technique was demonstrated in the context of Additive White Gaussian Channel (AWGN) and Rayleigh flat fading channel. However, the proposed technique should fail for multipath channel. In multipath scenario, the authors in [MLJ06a] proposed to combine OFDM modulation and IDMA system. Even so OFDM modulation is more sensitivity to carrier frequency offset (CFO) than single carrier modulation [TNA04]. The problem of CFO is defined as the mismatch in carrier frequency

between the transmitter and the receiver. The reason of this mismatch is the Doppler Effect. The presence of CFO in OFDM system causes the reduction of signal amplitude and the introduction of intercarrier interference from the other carriers. Thus a blind CFO estimation, wherein no additional sequence is added, was presented in [IHG09]. This estimation method, called CFO estimation-Syndrome Function Minimization (C-SFM), is based on a Maximum a Posteriori (MAP) approach. It consists to minimize the functions of the Log-Likelihood Ratios (LLR) of the syndrome elements. The syndrome is obtained according to the parity check matrix of the error correcting code. The principle of C-SFM was inspired by [IHD07] where the MAP approach was applied for blind frame synchronization. The study of blind synchronization technique in [IHG09] was used for single user transmission considering an AWGN channel and single carrier modulation. In this paper, we present the extension of this study when OFDM modulation and multipath channel are considered in the context of multiuser system. This paper is organised as follows: Section 1 describes the C-SFM algorithm for OFDM modulation in the context of single user transmission. In section 2, we adapt this algorithm to OFDM-IDMA multiuser system considering a flat fading channel and a quasi-static

Rayleigh fading multipath channel. Section 3 presents the simulations results of adaptation technique. Finally, Section 4 concludes the work.

## 1. Blind CFO estimation method for OFDM modulation in the context of single user transmission

We extend the blind CFO estimation algorithm proposed in [IHG09] for single carrier transmission to the context of OFDM is based on the calculation and the minimization of LLR function of the syndrome elements of the error correcting code. The spreading code used in [HSD06] for IDMA system was the repetition code. The authors in [ISH09] demonstrated that the method of blind synchronization can present a good performance when the syndrome elements are independents. Indeed, calculating the syndrome of each information block allows us to check whether a block corresponds to a valid codeword or not. The syndrome of each element is obtained according to the parity check matrix of the code. So, for codes having a sparse parity check matrix, the syndrome elements can be assumed independent. In our work, we consider a repetition code of rate  $(n_c - n_r)/n_c$  where the variables  $n_c$  and  $n_r$  represent respectively the length of a codeword and the number of parity relations. To validate the independence property of the syndrome, we use a repetition code whose  $G$  is the parity check matrix:

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

The bit informations are mapped by Binary Phase Shift keying (BPSK). Then, the real symbols  $x_k$  are divided in  $N_{\text{block}}$  OFDM blocks. An Inverse Discrete Fourier Transform (IDFT) is performed on each block to modulate the symbols using  $N_c$  orthogonal carriers. The IDFT output  $X_n$  is written as:

$$X_n = \sum_{k=0}^{N_c-1} x_k e^{j2\pi k \frac{n}{N_c}} \quad \forall k \in [0 \dots N_c - 1] \quad (1)$$

Adding a cyclic prefix (CP)  $\Delta$  before transmission and removing it in the receiver may avoid intersymbol interference (ISI) even in a scenario with severe multipath [TNA04]. We assume a perfect frame and timing synchronization in mono-user transmission. For each OFDM block, the received symbol  $R_n$  can be written as:

$$R_n = \sum_{l=0}^{L-1} h_l X_{n-l} e^{j2\pi(n-l)f_0} + w_n \quad \forall n \in [0 \dots N_c - 1] \quad (2)$$

where  $w_n$  represents a white complex Gaussian noise. The fading coefficients  $h_l$ , where  $l=0 \dots L-1$ , are constant during the OFDM symbol transmission.  $L$  is the number of paths which have an exponentially decaying profile. The frequency  $f_0$  represents the CFO which is uniformly distributed between  $-0.1/T_s$  and  $0.1/T_s$ . This carrier frequency offset is compensated by a frequency  $\tilde{f}$  after CP removal. Thus, the received signal becomes:

$$R_n(\tilde{f}) = R_n e^{-j2\pi n \tilde{f}} \quad \forall n \in [0 \dots N_c - 1] \quad (3)$$

An OFDM demodulation by Discrete Fourier Transform (DFT) operation is applied to the received signal  $R_n(\tilde{f})$ . The output of DFT operation  $r_k$  can be written as:

$$r_k = \sum_{n=0}^{N_c-1} R_n(\tilde{f}) e^{j2\pi k \frac{n}{N_c}} \quad \forall k \in [0 \dots N_c - 1] \quad (4)$$

Thereafter, we compute the cost function  $L_R(\tilde{f})$  corresponding to the real part of Log-Likelihood Ratios (LLR) of the syndrome [IHG09]. The cost function is expressed as:

$$L_R(\tilde{f}) = E \left[ \sum_{k=1}^{n_r} ((-1)^{u_k+1} \left( \prod_{j=1}^{u_k} \text{sign}(\Re(r_{k_j})) \right) \min_{j=1, \dots, u_k} |\Re(r_{k_j})| \right) \right] \quad (5)$$

Where  $u_k$  and  $k_j$  represent respectively the number of ones in the  $k^{\text{th}}$  row of the parity check matrix of the code and the position of the  $j^{\text{th}}$  non zero element in this  $k^{\text{th}}$  row. Through the variation of LLR according to  $\tilde{f}-f$ , we note that the LLR is minimal for  $\tilde{f}=f$ . Therefore, we can estimate the CFO by minimizing of LLR over  $\tilde{f}$ . However, the curve of  $L_R(\tilde{f})$  contains many local minima that are not random fluctuations. So, we propose to use the iterative Simulated Annealing algorithm presented in [IHG09] to find the global minimum of the system and estimate the CFO. The estimated CFO is:

$$\tilde{f}_{\text{est}} = \underset{\tilde{f}}{\text{argmin}} L_R(\tilde{f}) \quad (6)$$

The algorithm of Simulated Annealing is applied on the LLR function. At the final iteration, the CFO of the system can be estimated. The C-SFM performances in terms of Mean Squared Error (MSE) are presented in section 3. In section 2, we extend the C-SFM algorithm presented in this section for multiuser system OFDM-IDMA.

## 2. Blind CFO estimation for OFDM-IDMA system

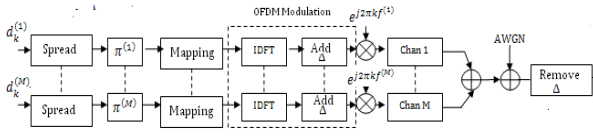
Before going further, let us first present the structure model of OFDM-IDMA. The only mean for user separation is to use different interleavers which are generated independently and randomly. The IDMA structure presented in [HSD06] was characterized by

an unique interleaver for each user. The separation was done by virtual different interleavers due to the de-synchronization of different users. At the transmitter shown in Fig. 1 for  $M$  simultaneous users, the information data from each user is first spread by a repetition code of rate  $R=1/6$ . Before mapping the resultant signal by BPSK modulation, this signal has to be interleaved by its unique interleaver. The real symbols of each user are divided in blocks. Then, an Inverse Discrete Fourier Transform (IDFT) is performed on each block to modulate the symbols using  $N_c$  orthogonal carriers for the transmission. A cyclic prefix  $\Delta$  is added before transmission. We assumed here a quasi-static Rayleigh fading multipath channel and perfect channel estimation. We considered the multipath channel features used in [MLJ06b]. The authors in [MLJ06a] studied the OFDM-IDMA system for quasi-static Rayleigh fading multipath channels assuming that all synchronizing parameters were correctly estimated. In our work, we consider the presence of CFO in OFDM-IDMA transmission system. The received signal  $R_n$  with mean  $E(R_n)$  can be written as:

$$R_n = \sum_{m=0}^{M-1} \sum_{l=0}^{L-1} h_l^m X_{n-l}^m e^{j2\pi(n-l)f^m} + w_n \quad (7)$$

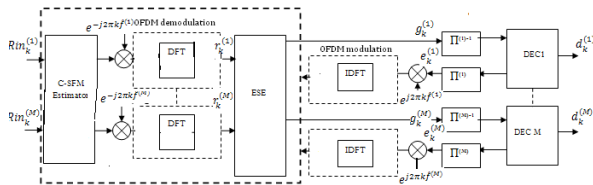
$$\forall n \in [0 \dots N_c - 1]$$

Where  $X_n^m$  represents the IDFT output of the user  $m$  and  $f^m$  is the CFO of this user. The coefficients fading of each user  $h_l^m$  were characterized in section 1.



**Figure 1.** Transmitter structure of an OFDM-IDMA scheme with  $M$  simultaneous users

In order to estimate the correct CFO of each user, we apply the blind CFO estimation method which was described in section 1. So, a blind estimator of carrier frequency offset, called C-SFM estimator, is added in the OFDM-IDMA receiver. The receiver structure of OFDM-IDMA with  $M$  simultaneous users is shown in Figure 2.



**Figure 2.** Receiver structure of OFDM-IDMA with  $M$  simultaneous users.

After removing the guard interval, the correction of CFO is applied before the OFDM demodulation which involves a Discrete Fourier Transform (DFT). In the

operation of estimation, we use the DFT to transform the symbols to the frequency domain in order to calculate the LLR as described in section 1. Then, we can compensate the CFO and demodulate the symbols to perform multiuser detection. After CFO estimation and compensation, the ESE only estimates and cancels the MAI. The iterative process includes both the OFDM modulation and OFDM demodulation. The operation of estimation and interference suppression is done successively for each user. In this receiver structure, a new ESE involved the CFO estimator is created. At each iteration, the operations on this new ESE and on the decoder DEC1 corresponding to the 1<sup>st</sup> user are firstly performed, then on those of 2<sup>nd</sup> user and so forth until user  $M$ .  $R_n^m$  represents the received signal of user  $m$  after compensation operation by his CFO. It is expressed by equation (11). We estimate the users interference MAI to remove it from this signal. To estimate the MAI, an IDFT operation is performed on the extrinsic information  $e_k^m$ . This latter is defined as a priori LLR at the ESE detector provided by the decoders. So the blind estimator input of user  $m$   $R_{in}^m$  contains only the user  $m$  contribution. So, using this signal to estimate the CFO will improve the performance of our CFO estimator. The ESE output is an extrinsic Logarithm Likelihood Ratio (LLR) of  $x_k^m$  denoted  $g_k^m$ . We assume that  $x_k^m$  is a Gaussian variable with mean  $E(x_k^m)$  and variance  $\text{Var}(x_k^m)$ .  $H_k^m$  are the coefficients of the channel frequency response. The DFT of  $w_n$  is a complex white Gaussian noise with variance  $\sigma^2$ .  $\text{Var}(r_k)$  is the variance of the received signal  $r_k$  after DFT operation. The algorithm of chip detection and CFO estimation, corresponding to OFDM-IDMA system, is summarized below, for a serial processing and BPSK modulation:

**Initialisation:**

$$e_k^m = 0 \quad \forall m, k$$

$$\forall m, k : E(x_k^m) = 0, \text{Var}(x_k^m) = 1$$

$$\forall m, n, R_n^m = R_n$$

$$\forall m : \tilde{f}^m = 0$$

$$\forall n, E(R_n) = 0$$

$$\forall k : \text{Var}(r_k) = \sigma^2 + \sum_{m=1}^M |H_k^m|^2$$

$$m=1, it=1$$

**Main iteration:**

– CFO estimation by C-SFM method:

The blind estimator input  $R_{in}^m$  is computed by:

$$R_{in}^m = R_n^m - e^{-j2\pi n l_{\text{prev}}^m} E(R_n) + E(X_n^m) \quad (8)$$

where  $E(X_n^m)$  corresponds to the IDFT of  $H_k^m E(x_k^m)$ . It is expressed by:

$$E(X_n^m) = \sum_{k=0}^{N_c-1} H_k^m E(x_k^m) e^{j2\pi k \frac{n}{N_c}} \quad \forall k \in [0 \dots N_c - 1] \quad (9)$$

The CFO estimated  $\tilde{f}^m$  is the output of the estimator.

The correct estimate of the CFO in the current iteration  $\tilde{f}_{\text{cur-it}}^m$  depends on a CFO of previous iteration  $\tilde{f}_{\text{prev-it}}^m$ . It is written as:

$$\tilde{f}_{\text{cur-it}}^m = \tilde{f}^m + \tilde{f}_{\text{prev-it}}^m \quad (10)$$

– The received signal for user  $m$  after CFO compensation is expressed as:

$$R_n^m = e^{-j2\pi n \tilde{f}_{\text{cur-it}}^m} R_n \quad (11)$$

– After OFDM demodulation, this signal becomes:

$$r_k^m = \sum_{n=0}^{N_c-1} R_n^m e^{-j2\pi k \frac{n}{N_c}} \quad \forall k \in [0 \dots N_c-1] \quad (12)$$

– So, the mean of this signal, after applying the DFT operation, is expressed by:

$$E(r_k^m) = \sum_{n=0}^{N_c-1} (e^{j2\pi n \tilde{f}_{\text{cur-it}}^m} E(R_n)) e^{-j2\pi k \frac{n}{N_c}} \quad \forall k \in [0 \dots N_c-1] \quad (13)$$

– LLR computation,  $\forall k$ :

$$g_k^m = 2 \times H_k^m \times \frac{r_k^m - E(r_k^m) + H_k^m E(x_k^m)}{\text{Var}(r_k^m) - (H_k^m)^2 \text{Var}(x_k^m)} \quad (14)$$

– De-interleaving, despreading, decoding, spreading, computation of the extrinsic information and interleaving are described in [LPL03].

– Current user's information update,  $\forall k$ :

$$E(x_k^m) = \tan\left(\frac{e_k^m}{2}\right) \quad (15)$$

$$\text{Var}(x_k^m) = 1 - (E(x_k^m))^2 \quad (16)$$

$$\Delta \text{Var}(x_k^m) = \text{Var}(x_k^m)|_{\text{cur-it}} - \text{Var}(x_k^m)|_{\text{prev-it}} \quad (17)$$

$$\text{Var}(r_k^m) = \text{Var}(r_k^m) + (H_k^m)^2 \Delta \text{Var}(x_k^m) \quad (18)$$

After the IDFT operation, the mean difference of the modulated signal between the current iteration and the previous iteration is expressed:

$$\Delta E(X_n^m) = e^{j2\pi n \tilde{f}_{\text{cur-it}}^m} E(X_n^m)|_{\text{cur-it}} - e^{j2\pi n \tilde{f}_{\text{prev-it}}^m} E(X_n^m)|_{\text{prev-it}} \quad \forall n \quad (19)$$

So, the mean of the received signal in this iteration is updated by:

$$E(R_n) = E(R_n) + \Delta E(X_n^m) \quad (20)$$

– If  $m=M$ , put  $m=1$  else, put  $m=m+1$  and back to equation (8).

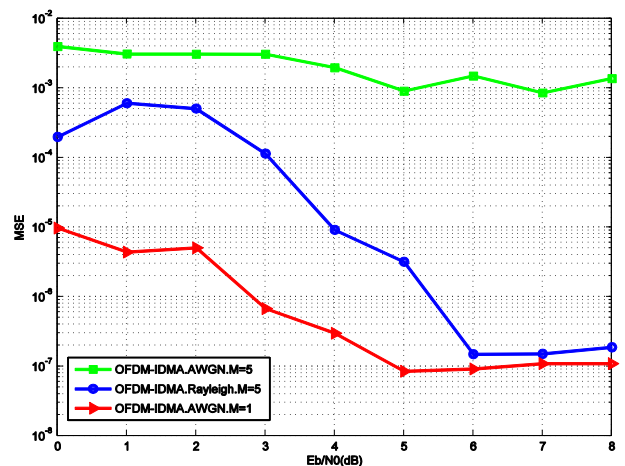
### 3. Simulation results

In our work, we analyze the performance of CFO estimation method when an OFDM modulation is considered in IDMA system. We plot the Mean Squared Error (MSE) curves versus  $E_b/N_0$  where we fix the number of iterations of Simulated Annealing to 1000. We assume a flat fading Rayleigh channel. OFDM parameters are given in Table 1 according to the 802.11a standard. The plotted curves are obtained by Monte Carlo simulations where a CFO is randomly chosen between  $-0.1/T_s$  and  $0.1/T_s$  in each run.

Sampling rate	$f_s = 20$ MHz
Useful symbol duration	$T_u = 3.2$ $\mu$ s
Guard interval duration	$T_g = 0.8$ $\mu$ s
Total symbol duration	$T_{\text{tot}} = 4.0$ $\mu$ s
Number of data sub-carriers	$N_c = 48$
FFT size	64

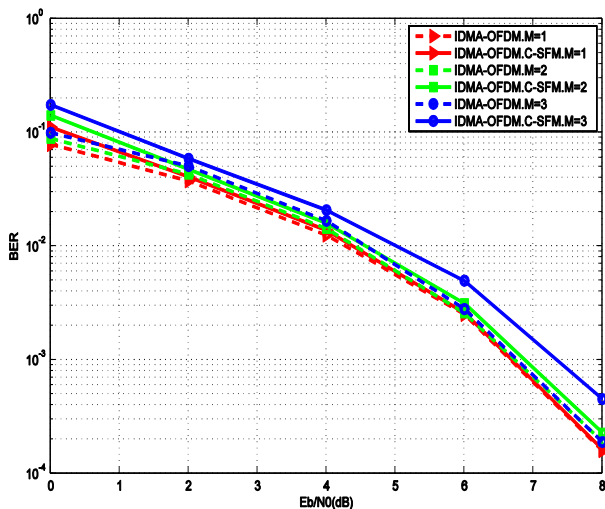
**Table 1.** OFDM system parameters from the 802.11a standard.

Figure 3 shows the performance of CFO estimation technique in OFDM-IDMA context. The curves are plotted for AWGN channel and flat fading Rayleigh channel. We consider 1 and 5 users in the transmission system. The blind CFO estimation is applied to a repetition code of rate  $R=1/6$  and having  $u_k=2$  non zero elements in each row of its parity check matrix. It is clear that when we increase the numbers of users, the performance of the CFO estimation technique is degraded. For a Rayleigh channel, the MSE are better than those for an AWGN channel. Indeed, as the proposed algorithm is based on a successive interference cancellation approach, we obtained better performance by starting with the most powerful user. Note that this is done naturally by the blind frame synchronization algorithm [HSD06].



**Figure 3.** MSE of blind CFO estimation in OFDM-IDMA context for AWGN channel and flat fading Rayleigh channel.

In order to evaluate the robustness of the C-SFM method in the context of OFDM-IDMA system for multi-path channel, we plot in figure 4 and figure 5 the Bit Error Rate (BER) curves assuming 1, 2 and 3 users for AWGN channel and 1 and 3 users for Rayleigh channel. The authors in [IHG09] demonstrated that the performance of blind CFO estimation technique is improved when we increase the number of iterations. Here, 3000 iterations of the Simulated Annealing are used to estimate the CFO. For the iterative receiver, 5 iterations are enough. We compare the performances of synchronous OFDM-IDMA with that asynchronous OFDM-IDMA using C-SFM technique. It is clear that the curves of one user for AWGN channel are almost similar from 2 dB. The curves of Rayleigh channel are stucked from 8 dB. We observe that when we increase the numbers of users, the gap between the curves of synchronous OFDM-IDMA and OFDM-IDMA using C-SFM technique becomes wider.



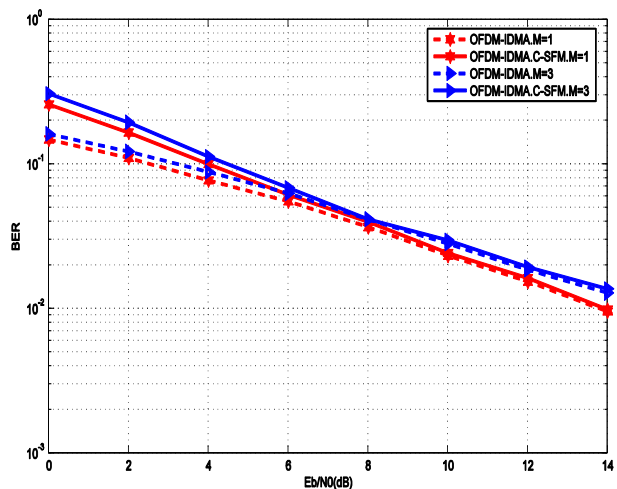
**Figure 4.** Asynchronous OFDM-IDMA system with blind CFO estimation C-SFM (full lines) versus synchronous OFDM-IDMA system (dashed lines), for  $M=1$  users, 2 users and 3 users and AWGN channel.

Figure 6 shows the same comparison when applying the C-SFM technique to OFDM-IDMA system considering a quasi-static Rayleigh fading multipath channel with 16 paths. Compared to the perfect synchronous system, using the blind CFO estimation method in OFDM-IDMA yields always better results for high  $E_b/N_0$ .

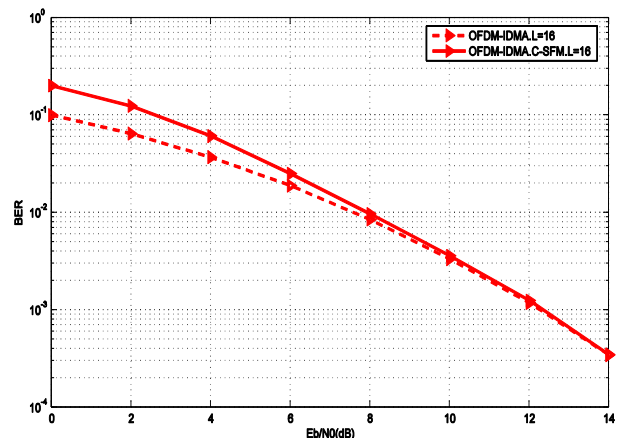
#### 4. Conclusion

We have proposed in this paper an adaptation technique of blind CFO estimation to a multi-users context using a multi-carrier modulation for quasi-static Rayleigh fading multipath. Applied to a repetition code having a sparse parity check matrix and a multi-users system which is based on different interleavers for user detection, the blind synchronization technique have presented a good performance for high  $E_b/N_0$ . The bit error rates of the OFDM-IDMA system with blind CFO estimation have been almost the same as the ones obtained in the

perfect synchronization case. Perfect recovery of the carrier frequency offset has been achieved for high  $E_b/N_0$ . Furthermore, simulated results have shown that there is performance degradation when the number of users increases.



**Figure 5.** Asynchronous OFDM-IDMA system with blind CFO estimation C-SFM (full lines) versus synchronous OFDM-IDMA system (dashed lines), for  $M=1$  users and 3 users and Rayleigh channel with  $L=1$ .



**Figure 6.** Asynchronous OFDM-IDMA system with blind CFO estimation C-SFM (full lines) versus synchronous OFDM-IDMA system (dashed lines), for  $M=1$  and quasi-static Rayleigh fading multipath channel with  $L=16$ .

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