Asymmetrical Stripline Based Method for the Electromagnetic Characterization of Metamaterials

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Abstract—An experimental method for obtaining the effective electromagnetic parameters of metamaterials is presented. The measurement cell consists in an asymmetric stripline which satisfies certain conditions required for the characterization of this type of materials. The advantages of this cell, its electromagnetic analysis and preliminary experimental and simulated results are shown.

1. INTRODUCTION
The metamaterials are artificial composite structures that have electromagnetic properties not observed in their constituent materials or in nature. The most important characteristic is that it is possible to control the response of their permittivity and permeability and adapting their behavior to specific applications. These structures are used in applications involving antennas, filters and couplers due to the possibility of miniaturization which is made by adjusting the value of the refractive index.

The most common way to fabricate this type of materials is by the periodic inclusion of metallic wires inside a dielectric. The shape of these inclusions usually takes the form of Split Ring Resonators (SRR) [1]. The interaction between the electromagnetic fields and the metallic strips produces a resonant behavior that combines two effects: a capacitive response due to the presence of gaps between wires and an inductive effect due to the interaction of the magnetic field with the ring shaped strips. The combination of these two effects produces the enhanced permittivity and permeability and could even result in negative values of these quantities.

Other characteristics of metamaterials are the dispersion, the anisotropy and the heterogeneity. The dispersion: frequency dependence of the permittivity and permeability, implies that in order to describe its behavior a wide band analysis is required. The anisotropy means that the electromagnetic properties of the material are directionally dependent. Finally, due to the heterogeneous character of metamaterials it is necessary to determine if it is possible to utilize homogenization methods for extracting the effective parameters that represent all the structure.

This article presents a new broadband method applied in the characterization of inclusion-based metamaterials SRR type, metasolenoid or similar. It uses an asymmetrical stripline that has been developed in the laboratory for the characterization of magnetic materials [2, 3]. The configuration of the electromagnetic fields in the region where the sample is placed provides an appropriated excitation to the material. Two types of electromagnetic analysis of the stripline are presented. Measured and simulated results for a metasolenoid-type metamaterial are shown.

2. CHARACTERIZATION METHODS
The characterization of the electromagnetic properties of metamaterials is a significant stage in the process of developing new applications and new type of structures. Many factors must be considered for the characterization of metamaterials, as the frequency band, the geometry and the orientation and distribution of the metallic inclusions. This last point is fundamental because the response of the material is determined by the way the electromagnetic field interacts with the inclusions. The enhanced permittivity and permeability result only when the electric and magnetic fields are oriented in one specific direction. In the particular case of a metamaterial type SRR or similar, the enhanced response occurs when the direction of the magnetic field is normal to the surface of the structure, and the electric field is tangent to the spirals. The magnitude of both fields must be constant over all the volume of the metamaterial. The characterization procedure has to provide the correct polarization and distribution of the EM field and take into account the geometry and the orientation of the sample inclusions.

There are some experimental procedures already used for extracting the constitutive parameters of metamaterials. The main objective of these methods is to obtain the S-parameters of a structure charged of material and then determining the permittivity and permeability using different analytical inverse procedures. Free space methods allow us to perform measurements in high frequencies.
They are not recommended for frequencies below 10 GHz, because big samples and different setups are required depending on the frequency band of interest [4]. Other methods based on resonant cavities provide accurate results, but only at a specific frequency, which is not convenient because of the dispersive nature of the metamaterials [5, 6]. The transmission line based methods, such as waveguide [7] or microstrip line [8] could be used in a wide frequency band and do not require big samples for frequencies above 3 GHz, but their main disadvantage is that they do not present the field distribution that produces the expected behavior of the metamaterial. A coaxial line is not practical for these cases because it is necessary to adapt the geometry of the metamaterial to a cylindrical shape.

3. STRIPLINE METHOD

The method proposed in this work uses an asymmetrical stripline that has been developed in our laboratory for the broadband characterization of magnetic materials [2, 3]. The configuration of electromagnetic fields is well adapted for metamaterials characterization. In-situ characterization is allowed because it reproduces an electromagnetic environment close to the one met in practice (use of metamaterial in planar technologies). The measurement cell is composed by a central conductor and two ground planes (Figure 1). The conductor strip is closer to the inferior ground plane to concentrate the most part of the energy in this region where the sample is set. In this area the electric field is parallel to the inclusions and the magnetic field is perpendicular to them. The amplitude of both fields is constant for the entire sample and it is possible to consider the propagation of a quasi-TEM mode. To ensure homogeneous field distribution to avoid the edge effects in the particular case of metamaterials, the width of the sample should not exceed the width of the central conductor. This is why the cell could not be compared to a real microstrip line. The Figure 1 present the electromagnetic field distribution in the cell containing a sample of material.

Using a Vectorial Network Analyzer the $S$-parameters of the propagation structure are measured. The effective parameters of the material are obtained from this measurements after an electromagnetic analysis of the cell. We use two different theoretical approaches for calculating $\epsilon_{\text{eff}}$ and $\mu_{\text{eff}}$ based on the quasi-static approximation.

The first one is a variational procedure in which the cross section of the cell is divided in horizontal layers (air, material and air gaps) representing the different media of the structure (Figure 2(c)). The variational theory consist in homogenizing the transversal section of the line and representing it with an effective permeability and permittivity (Figure 2(b)). Using Green’s functions it is possible to extract the capacitance and inductance per unit length of the line and the theoretical values of the effective parameters could be represented in terms of the loaded and unloaded C and L. This first stage corresponds to the direct problem. Using Nicolson/Ross technique it is possible to extract the $\epsilon_{\text{eff}}$ and $\mu_{\text{eff}}$ from the reflection and transmission coefficient of the cell ($S$-parameters). Then, for obtaining the intrinsic parameters of the material from the measurements, an inverse problem is applied. The $\epsilon$ and $\mu$ from de sample are obtained by an optimization procedure that matches the theoretical and measured values.

This model considers that the length of the sample is infinite in the transversal direction. This assumption could not be applied in the case of metamaterials since the configuration of the electromagnetic fields in the region which is not covered by the central strip is not appropriated for metamaterials's measurement (Figure 2(a)). A correction that allows to obtain the constitutive parameters of the metamaterial should be applied as an additional step for this method.

Figure 1: Asymmetrical measurement cell.
The second approach is a quasi-static method based in transmission line theory. Due to the geometry of the cell (concentration of energy below the central conductor), the cross section could be represented as a two conductor structure with boundary conditions as shown in Figure 2(d). We can assume then that only the quasi-TEM mode is propagated. The transmission line analysis allows to represent the S-parameters in terms of the capacitance and inductance per unit length of the line. The inverse problem gives an expression that directly relates the values of the effective permeability and permittivity to the reflection and transmission parameters and no optimization procedure is needed \[2\].

There is still a correction that must be done, due to the presence of air gaps appearing between the sample and the ground plane and between the metamaterial and the central conductor. For this correction an Effective Medium Approximation (EMA — Wiener’s law) is applied. The final expressions are given in Equations (1) and (2).

\[
\mu_r = \frac{\omega \mu_0 h_a (1 - R) - Z_0 W / (\gamma)(1 + R)}{\omega h_a \mu_0 (R - 1)},
\]

(1)

\[
\epsilon_r = \frac{h_s (\gamma^2)}{\omega^2 \mu_0 \epsilon_0 (h_s \mu_r + h_a) - h_a \gamma^2},
\]

(2)

where \(\omega\) is the angular frequency, \(h_a\) is the height of the air gap, \(h_s\) is the height of the sample, \(Z_0\) is the characteristic impedance of the cell, \(R\) is the reflection coefficient, \(\gamma\) is the propagation constant and \(W\) is the width of the strip conductor.

The last step to retrieve the intrinsic values, taking into account the air over the central conductor is then achieved using the microstrip line model proposed by Hammerstad.

4. ELECTROMAGNETIC SIMULATIONS

Electromagnetic simulations of the measured metasolenoid were made using the commercial software Ansoft HFSS FEM-based simulator (Figure 3(b)). The conditions that must be considered in the simulation of metamaterials are the same mentioned before for measurements: A correct polarization of electromagnetic fields and constant amplitude over all the volume of the metamaterial. Also in this case it is important to define the appropriated boundary conditions and exciting ports such as an infinite periodic structure is represented.

One slab of material was considered. The boundary conditions applied were periodic electric and magnetic conductors (PEC and PMC), and wave ports were used as excitations (Incident TEM mode). This first simulation gave us a general idea of how is the frequency response of the metamaterial, and applying Nicolson Ross expressions obtained from \[9\] it was possible to obtain the permittivity and permeability from the S-parameters.

5. RESULTS

Figure 3(a) presents the measurement cell and the sample of metamaterial. Figure 4 compares the permittivity and permeability values obtained for the sample using the stripline measurement cell from the two retrieval procedures (variational and transmission line analysis). The theoretical study and the measurements were performed between 10 MHz and 6 GHz. The frequency response of the metamaterial was completely characterized and it was possible to observe the resonant behavior of the electric and magnetic parameters. Also the two retrieval procedures show good agreement and this can be confirmed comparing with the results obtained from the FEM-based electromagnetic simulation. The two different values of permittivity obtained from the TL analysis show the utility of applying mixing laws and effective models due to the heterogeneous nature of the cross section.
of the cell. Taking into account the presence of air gaps allow us to extract the intrinsic values and to approach the results obtained with the two analysis methods and the FEM-based simulations.

6. CONCLUSION
This work presents the extension of a characterization method applied to the representative measurement of the permittivity and permeability of metamaterials. The complex electromagnetic parameters $\epsilon$ and $\mu$ of a sample were obtained measuring the $S$-parameters of an asymmetrical stripline measurement cell and applying analytical procedures. The results were validated applying two different analysis of the cell and comparing the measured data with accurate electromagnetic simulations.

REFERENCES

