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# MODEL-ORDER REDUCTION OF VLSI CIRCUIT INTERCONNECTS VIA A LAGUERRE REPRESENTATION

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## Abstract

The VLSI (Very Large Scale Integration) industry has the tendency to decrease circuit size, increase speed, assuring ever lower energy consumption and ever higher integration density of analogical components accompanied by digital blocs. With this tendency circuit designers are faced with a new challenge: the analysis and modeling of logical and analogical signals propagating between two circuit points. The search for high speed applications makes the effects of interconnects, usually neglected in the past, an important issue; noise, delay, distortion, reflections and cross talk are just some of these effects. High integration density, miniaturization, high working frequencies are three great factors which prevent interconnects to be considered small independent circuits. Thus, simulation becomes a rather difficult task [4]. Still, wouldn't it be possible to replace complete interconnect equivalent circuits by simpler and more flexible models. Our new model-order reduction technique is mainly based on the use of the Laguerre representation [1], [5] and a simple operator used to generate an approximation base [2].

## I - Laguerre Representation

The method chosen for this application benefits from recent results on the modeling of irrational Laplace transformations [1]. It is based on a Laguerre description of the system and uses a simple linear operator. Any function  $\hat{f}(s)$  can be represented as a Laguerre series  $f_n$  such as

$$\hat{f}(s) = \sum_{n=0}^{\infty} f_n \hat{\phi}_n(s) \quad (1)$$

$F(z)$ , the z-transform of Laguerre coefficients  $\{f_n\}_{n \geq 0}$  may be calculated after performing the variable change:  $z = (s - \alpha)/(s + \alpha)$ . It yields

$$f(s) \longrightarrow F(z) \triangleq \sum f_n z^{-n} = 2\alpha \frac{z}{z-1} \hat{f}\left(\alpha \frac{z+1}{z-1}\right) \quad (2)$$

We then build an array of transfer functions  $\Omega_r = \{\hat{f}_0, \hat{f}_1, \dots, \hat{f}_r\}$  obtained by successive applications of the operator denoted as  $\Lambda$  and defined by :

$$\begin{aligned} \hat{f}_0(s) &= \hat{f}(s), \\ \hat{f}_{i+1}(s) &\triangleq \Lambda \hat{f}_i(s) = \left[ \hat{f}_i(s) - (2\alpha)/(s + \alpha) f_i(\alpha) \right] (s + \alpha)/(s - \alpha) \end{aligned} \quad (3)$$

We denote as  $\{f_{i,n}\}_{n \geq 0}$  the Laguerre coefficients of the  $\hat{f}_i(s)$  function and as  $\Theta_r = \{F_0, F_1, \dots, F_r\}$  the array of the respective z-transforms obtained by applying (2). We can easily demonstrate that

$$F_{i+1}(z) = [F_i(z) - f_{i,0}]z \quad (4)$$

where  $F_0(z) = F(z)$ . The Laguerre spectrum (the coefficient array) of  $\hat{f}_{i+1}(s)$  is obtained in a quite obvious manner from that of  $\hat{f}_i(s)$  and so on

$$f_{i+1,n} = f_{i,n+1} = \dots = f_{0,i+n+1} \quad (5)$$

for any  $n \geq 0, i \geq 0$ . There is no need to recalculate the Laguerre coefficients that describe each of the  $\hat{f}_i(s)$  functions which makes the operator  $\Lambda$  very easy to use. We can also demonstrate that the  $\Lambda$  operator preserves the position of poles. This is an important property in terms of reduced model stability. As a consequence the array  $\Omega_r$  represents a basis of approximation functions that efficiently determines the reduced order model of  $\hat{f}(s)$ . Let  $\Psi$  denote the Gram matrix of inner products  $\psi_{i,j} = \langle \hat{f}_i, \hat{f}_j \rangle$  for  $i, j = 0, 1, \dots, r-1$  and  $\bar{b}$  the vector  $[\psi_{0,r}, \psi_{1,r}, \dots, \psi_{r-1,r}]^T$  where  $^T$  denotes the transpose and  $r$  the order of the reduced model. The best  $r$  order  $\bar{a} = [a_0, a_1, \dots, a_{r-1}]^T$  vector which minimizes the mean square error (MSE) is the solution to the system of equations given by:

$$\Psi \bar{a} = -\bar{b} \quad (6)$$

The denominator coefficients are easily obtained from  $\bar{a}$  [2]. The inner products  $\psi_{i,j}$  are expressed exclusively as a function of the Laguerre spectrum of  $\hat{f}(s)$  given by a simple sum:

$$\psi_{i,j} = \sum_{n=0}^{\infty} f_{n+i} f_{n+j} \quad (7)$$

Equation (6) yields a sub-optimal numerator but a numerator optimization can be performed in order to minimize MSE without rendering the procedure too time consuming [6].

## II – Reduction method applied to interconnects

### II.1. Physical interconnect model

In order to test our method, we first apply it in a very simple case (Fig 1): that of a single line whose step response is simulated.

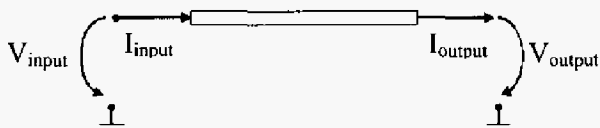


Figure 1 : example of single line

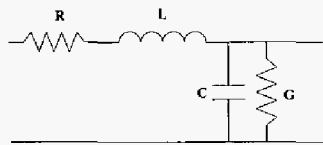


Figure 2 : equivalent circuit

In this elementary case, we denote the impedance and respectively admittance terms as  $Z = R + jL\omega$  and  $Y = G + jC\omega$ . When structure size is comparable to wave length, the simulation of propagation and diffraction phenomena may rest upon a discrete volume representation (finite elements, finite differences). With a finite element method, for example, a complex (high order) system must be solved for each frequency which usually proves to be time consuming. Our work is rather oriented towards reduced order models, susceptible of allowing fast system analysis over a wide frequency band. The choice of a simple structure allows a complete simulation of the device, as close to physical reality as possible; propagation constants and line parameters are obtained by electromagnetic analysis. Of this representation we extract the admittance matrix.

$$\begin{pmatrix} i_{input} \\ i_{output} \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} v_{input} \\ v_{output} \end{pmatrix} \quad (8)$$

where  $Y_{11} = Y_{22} = \frac{ch(\sqrt{ZY})}{Z(\sqrt{ZY})^{-1}sh(\sqrt{ZY})}$

and  $Y_{12} = Y_{21} = -\frac{1}{Z(\sqrt{ZY})^{-1}sh(\sqrt{ZY})}$

Each element of the matrix (8) can be considered an irrational transfer function, in other words a function of infinite order which we seek to reduce. The delay present in  $Y_{12}$  makes it very difficult to reduce to a reasonably small order simply

using the above method. In the next paragraph, we suggest an iterative alternative. This preliminary work will serve as a reference for future simulations using simplified models.

### II.2. Iterative Reduction Method

Given the physical reality behind the admittance functions it is quite obvious that a delay will be present in the trans-admittance terms ( $Y_{12}=Y_{21}$ ). Reduction methods have difficulties dealing with a pure delay, especially when relatively small output orders are needed.

In order to achieve this goal one idea would be to extract the delay, perform the reduction for the non-delayed signal and then reinsert the delay [7], [4]. This approach however is not desirable if we are to find an equivalent physical circuit for the reduced model.

A solution allowing us to obtain small order models including delays relies upon an unexpected property of the  $\Lambda$  operator. The iterative application of this operator on a function, with a gradual decrease in order, actually improves its efficiency, especially for signals containing delays (see chapter III).

For example: rather than search for an order 8 reduced model of the original irrational function, we will search for an order 20 and then approximate this order 20 by an order 8.

Experiments show that two up to four iterations can make a considerable difference in terms of MSE without drastically increasing computation time. Also, once a successful succession of steps is found for one signal, it can be reused with good results for similar signals.

## III Results

To illustrate the method's efficiency in dealing with delays, we have chosen the example of a rather long line ( $l=10mm$ ). The admittance matrix is  $2 \times 2$  and symmetrical, its elements being of infinite order. When we consider  $Y_{11}$  and  $Y_{12}$  as transfer functions, the step responses of the original system and of an order 4 model are given below. The order 4 responses, obtained using the direct method, are represented in figures 4 and 5.

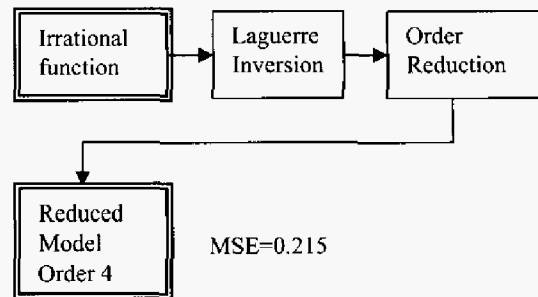


Figure 3: direct method

As we can see in Fig. 4, the order 4 model obtained by direct reduction of the irrational function (see Fig. 3) does not provide a satisfactory model of  $Y_{12}$ . This is due to the great delay.

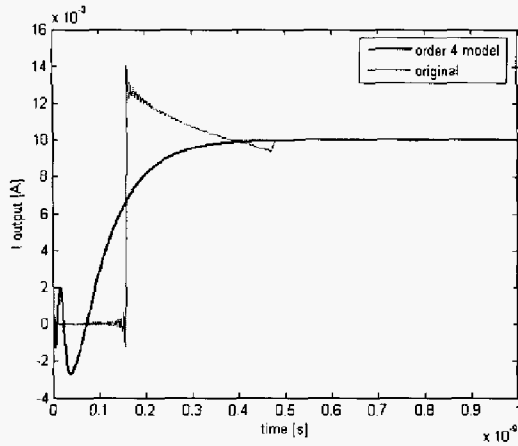


Figure 4: step response of  $Y_{12}$ , direct reduction

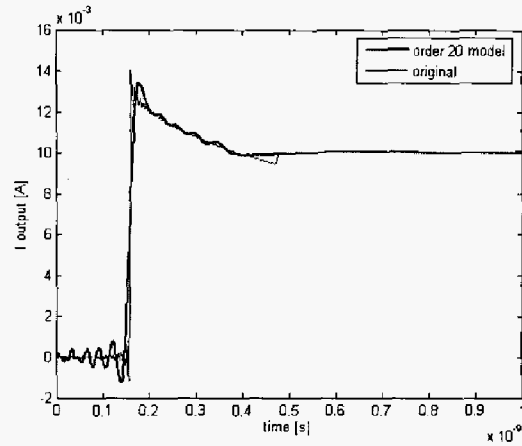


Figure 7: step response of  $Y_{12}$  first iteration – order 20

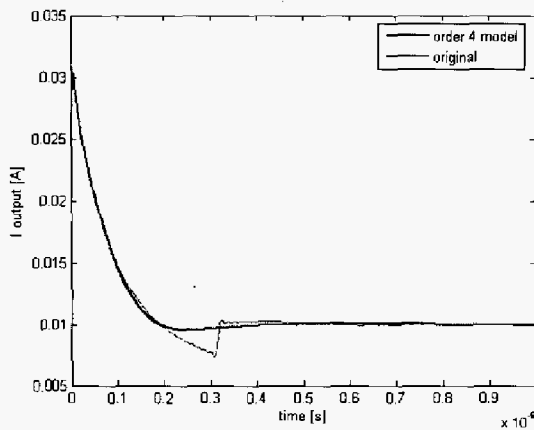


Figure 5: step response of  $Y_{11}$ , direct reduction

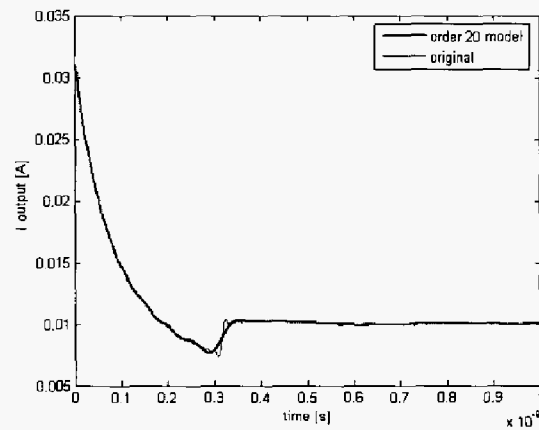


Figure 8: step response of  $Y_{11}$  first iteration – order 20

For a better approximation, we employ the iterative method whose principle is illustrated in Fig.6. The method is initialized with an order 20 model. Four iterations are sufficient to obtain the order 4 model illustrated in Fig.9 and Fig.10.

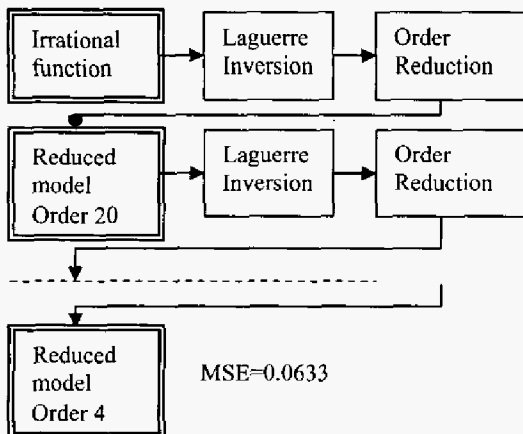


Figure 6: iterative method

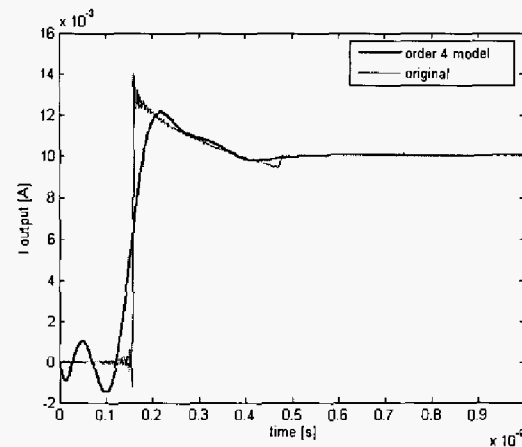


Figure 9: step response of  $Y_{12}$ , order 4 after 4 iterations

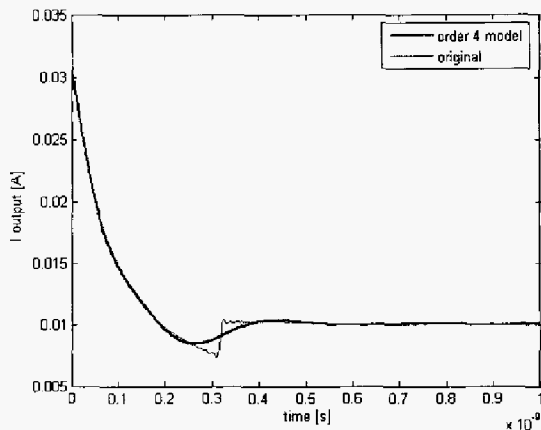


Figure 10: step response of  $Y_{11}$ , order 4 after 4 iterations

The initial order 20 approximation gives a first mathematical approximation of the delay. This initial model is then easier to reduce.

### Conclusions

The new model-order reduction technique presented here is essentially based on the use of a simple linear operator applied to a Laguerre representation in order to create a basis of approximation functions. The properties of Laguerre functions and the simplicity of the operator ease the construction of the basis. The stability of the original system is preserved. In the context of interconnection modeling this operator is applied to the admittance matrix. The perspectives of our work would be to synthesize a simple equivalent circuit from the reduced model and to extend our method to more complex structures.

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