Modulation Recognition for MIMO Communications
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Abstract: The blind recognition of communication parameters is an important research topic in both commercial and civilian systems. In this paper, we investigate the blind recognition of the modulation. Currently most part of the existing algorithms assumes that the transmitter uses a single-antenna. This study extends the problem for multiple-antennas (MIMO) systems. We adopt a Maximum Likelihood approach for the blind recognition of the modulation and we consider two different situations. First, we assume the channel knowledge at the receiver side and we expose the optimal solution which is called Average Likelihood Ratio Test (ALRT). Then, we relax this assumption and we propose a second method based on a Hybrid Likelihood Ratio Test (HLRT).

Keywords: MIMO, Spatial Multiplexing, Modulation Recognition, Electronic Warfare, Cognitive-Radio.

1. Introduction

With the integration of Internet and multimedia applications in next generation wireless communications, the demand for wide-band high data rate and robust communication services is growing. To meet these requirements, one of the most promising technologies relies on the use of multiple-antennas at both the transmitter and receiver side [1]. These systems, called MIMO, can be divided into two categories. On the one hand, MIMO communications with Spatial Multiplexing (SM) achieve high data rates by transmitting independent and separately encoded data signals from each of the multiple transmit antennas. On the other hand, MIMO with Space-Time Coding (STC) improves the
robustness of the communication by transmitting space-time redundancy [2]. Due to their high performances, MIMO-SM and MIMO-STC systems would play a key role for the development of the next generation wireless communications.

In a non-cooperative context, the blind recognition of MIMO communications parameters is a new challenging problem. Before decoding the received signals, the receiver needs to blindly estimate the number of transmitter antennas, the space-time coding, the channel and the modulation. Different approaches have been exposed in literature for the blind estimation of the number of transmitter antennas; a review is exposed in reference [3]. Furthermore algorithms devoted to the recognition of the Space-Time coding [4, 5, 6] and the estimation of the channel matrix [7, 8, 9, 10] are also available. Concerning the blind recognition of the modulation, several approaches have been proposed for Single-Input Single-Output communications (SISO) systems [11, 12] but they are not suitable for MIMO systems.

In this study, we propose a solution for the blind recognition of the modulation in MIMO-SM systems. Section 2 presents the signal model and the assumptions. In Section 3, we develop the optimal solution under the ideal case i.e., when the channel is known at the receiver side. Then, in Section 4, we adapt this solution for the non-ideal case i.e., when the channel knowledge is unknown. Finally, the performances of two proposed methods are compared in Section 5.

2. Signal Model and Assumptions

Let us consider a MIMO-SM transmitter using $nt$ antennas and a receiver composed of $nr$ antennas. Under the assumption that the channel is frequency-flat and time-invariant, the $k^{th}$ received samples, denoted $Y(k)=[y_1(k) \ldots y_{nr}(k)]^T$, can be expressed as:

$$Y(k) = HS(k) + B(k),$$  \hspace{1cm} (1)

where:

- the column vector $B(k)=[b_1(k) \ldots b_{nr}(k)]^T$ of size $nr$ corresponds to the noise samples. This random vector is assumed to be a spatially-white circular Gaussian random with zero mean and variance $\sigma^2$ i.e., $B(k) \sim N(0,\sigma^2 I_{nr})$, where $I_{nr}$ denotes the identity matrix of size $nr$;

- the elements of the $nr \times nt$ matrix $H$ correspond to the complex channel gain between the transmit and receive antennas. We assume that the number of receivers is greater than the number of transmitter i.e., $nr \geq nt$;
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– the column vector \( S(k) = [s_1(k) \ldots s_{nt}(k)]^T \) of size \( nt \) corresponds to the \( k^{th} \) transmitted symbols. These symbols are assumed independent and identically distributed. Furthermore, without loss of generality, we consider that the energy of the transmitted symbols is normalized i.e., \( E\left[|s_u(k)|^2\right] = 1 \) and that the gain factor is contained into the channel coefficients.

The goal of this paper is to blindly recognize the modulation format of the transmitted signals, denoted \( \Omega \), from \( N \) received samples \( Y = [Y(1), \ldots, Y(N)] \).

We restrict our study to the general class of linear and memoryless modulations; these modulations are described in the reference [13]. Furthermore, we assume that the number of transmitters and the noise variance are known at the receiver side. In a non-cooperative context, several approaches have been proposed in literature to estimate \( nt \) and \( \sigma^2 \) from the covariance matrix of the received samples \( E\left[YY^H(k)\right] \); a review of these methods is available in reference [3].

3. Modulation Recognition with Channel Knowledge

Within the Likelihood-based framework, the modulation recognition is formulated as a multiple composite hypothesis-testing problem. Let us denote by \( \Delta[Y/\Omega, H] \) the Likelihood Function (LF) of the modulation \( \Omega \). The selected modulation, denoted \( \hat{\Omega} \), is the one which maximizes the Likelihood Function, i.e.:

\[
\hat{\Omega} = \arg \max_{\Omega_u \in \Theta} \left( \Delta[Y/\Omega_u, H] \right),
\]

where \( \Theta = \{\Omega_1, \ldots, \Omega_p\} \) denotes the set of all possible modulations. The Likelihood Function depends on two random parameters: the transmitted symbols and the Gaussian noise. To take into account these random parameters, the optimal solution in the Bayesian sense is the Average Likelihood Ratio test (ALRT) [12], which is defined by:

\[
\Delta[Y/\Omega_u, H] = \int_{\text{S}} \Delta[Y/\Omega_u, S, H] P[S/\Omega_u] \, dS,
\]

where the matrix \( S = [S(1), \ldots, S(N)] \) of size \( nt \times N \) denotes the unknown transmitted symbols and where \( P[S/\Omega_u] \) corresponds to the probability of \( S \) given the modulation \( \Omega_u \). Using the fact that the transmitted symbols are independent and identically distributed, Equation (3) can be expressed as:

\[
\Delta[Y/\Omega_u, H] = \prod_{k=1}^{N} \int_{S(k)} \Delta[Y(k)/\Omega_u, S(k), H] P[S(k)/\Omega_u] \, dS,
\]

where \( S(k) \) denotes the column vector of size \( nt \) corresponding to the \( k^{th} \) transmitted symbol.
where the vector $S(k)$, of size $nt$, denotes the unknown transmitted symbols at sampling time $k$, and where $P[S(k)/\Omega_u]$ corresponds to the probability of $S(k)$ given $\Omega_u$. Let us denote by $M_u$ the number of states of the constellation $\Omega_u$. As the transmitted symbols are independent and identically distributed, the probability $P[S(k)/\Omega_u]$ is equal to $\frac{1}{(M_u)^u}$.

Using this result in Equation (4) leads to:

$$
\Delta[Y/\Omega_u, H] = \frac{1}{(M_u)^N} \prod_{k=1}^N \sum_{S(k) \in \Omega_u^n} \Delta[Y(k)/S(k), H].
$$

The Likelihood Function $\Delta[Y(k)/S(k), H]$ is equal to the probability density function of $Y(k)$ given the transmitted symbol $S(k)$, the channel matrix $H$ and the noise variance $\sigma^2$. From Equation (1), we obtain:

$$
\Delta[Y(k)/S(k), H] = \frac{1}{(\pi\sigma^2)^u} \exp\left[ -\frac{1}{\sigma^2} \|Y(k) - HS(k)\|_F^2 \right],
$$

where $\|\cdot\|_F$ corresponds to the Frobenius norm. Finally, using Equation (6) in (5) leads to:

$$
\Delta[Y/\Omega_u, H] = \frac{1}{(M_u)^N} \prod_{k=1}^N \sum_{S(k) \in \Omega_u^n} \exp\left[ -\frac{1}{\sigma^2} \|Y(k) - HS(k)\|_F^2 \right].
$$

The optimal solution to the modulation recognition problem is to maximise Equation (7) with respect to $\Omega_u$, which is equivalent to maximise the log-Likelihood Function:

$$
\log(\Delta[Y/\Omega_u, H]) = -N \cdot nt \log(M_u) - nr \log(\pi\sigma^2) +
$$

$$
+ \sum_{k=1}^N \log \left( \sum_{S(k) \in \Omega_u^n} \exp\left[ -\frac{\|Y(k) - HS(k)\|^2_F}{\sigma^2} \right] \right).
$$

Let us focus on Equation (8). One can note that the Likelihood Function is composed of several sums over the set $(\Omega_u)^n$. As this set is composed of $(M_u)^n$ elements, the computation of the LF can be difficult when the number of states, $M_u$, or the number of transmitters, $nt$, is large. Furthermore, the computation of the log-Likelihood Function requires the knowledge of the channel matrix. This information is usually unknown in a non-cooperative environment. To overcome this second problem, we propose a sub-optimal approach in the next section.
4. Modulation Recognition without Channel Knowledge

In a non-cooperative context, the channel matrix is unknown at the receiver side. A logical solution to approximate the log-Likelihood Function $\Delta[Y/\Omega_m, H]$ is to estimate $H$, under the assumption that the transmitted symbols belong to the constellation $\Omega_m$, and to use this estimate in the ALRT function. This approach is called Hybrid Likelihood Ratio Test (HLRT). Our proposed HLRT algorithm is described in Figure 1.

First, an initial channel estimate is obtained with an Independent Component Analysis [14]. Then, we use the modulation assumption to apply a phase correction algorithm. Finally, the channel estimate is used in Equation (8) to compute the log-Likelihood Function of each modulation. The first step of the algorithm does not depend on the modulation assumption while the 2nd and 3rd steps depend on it. The following subsections detail each step.

4.1 ICA algorithm

The Independent Component Analysis (ICA) is a computational method for separating a multivariate signal into additive components supposing the mutual statistical independence of the non-Gaussian source signals. A review of ICA algorithms is exposed in reference [14]. By setting the number of independent components equals to the number of transmitter antennas $nt$ with $nt \leq nr$, an ICA algorithm permits us to estimate the channel matrix up to a permutation and a phase ambiguity. In our study, we use the JADE algorithm which is based on the Joint Approximate Diagonalisation of Eigen-matrices [15]. Let us denote by $\hat{H}$ the mixing matrix estimated with JADE. Assuming a perfect separation, the channel can be expressed as:

$$H = \hat{H}P,$$

where $P$ is the permutation factor and $D$ is a diagonal matrix which contains the phase ambiguities, i.e.:

$$D = \begin{pmatrix}
    e^{j\theta_1} & \cdots & 0 \\
    \vdots & \ddots & \vdots \\
    0 & \cdots & e^{j\theta_{nt}}
\end{pmatrix}.$$

Figure 1. Approximation of the Likelihood Function $\Delta[Y/\Omega_m, H]$
Theorem 1: When the transmitted symbols belong to the same constellation, the permutation factor does not modify the value of the log-Likelihood Function, i.e.:

$$\log\left(\Delta[Y/\Omega_u, H]\right) = \log\left(\Delta[Y/\Omega_u, \hat{H}D]\right). \quad (11)$$

**Proof:** The log-Likelihood Function is given by Equation (8) with $H = \hat{H}D$. Let us focus on the terms:

$$\sum_{S(k)\in\Omega_u^n} \exp\left[-\frac{1}{\sigma^2} \left\| Y(k) - \hat{H}DPS(k) \right\|^2_F \right]. \quad (12)$$

As the transmitted symbols in each antenna belong to the same modulation, $S(k)$ and $PS(k)$ belong to same set $\Omega_u^n$. So the sum can be directly computed by setting $P$ equals to $I_n$. This operation only permutes the element of the sum and, as the sum is commutative, it does not change the final result.

Theorem 1 states that the permutation matrix does not change the value of the Likelihood Function. Nevertheless, the unknown phase matrix $D$ affects the LF function. We propose to estimate this phase matrix in **Subsection 4.2**.

**4.2 Phase correction algorithm**

In the second stage, we exploit the modulation assumption to correct the phase ambiguities. Under the assumption that the symbols belong to the modulation $\Omega_u$, the unknown phase factor can be estimated by using the constellations property of $u$. First, let us consider the separated components, $Y^s(k) = [y^s_1(k) \ldots y^s_m(k)]^T$, given by:

$$Y^s(k) = \hat{H}^{-1}Y(k), \quad (13)$$

$$Y^s(k) = DPS(k) + \hat{H}^{-1}B(k). \quad (14)$$

Using Equations (11) and (13), each separated component can be expressed in a vector form as:

$$y^S_u(k) = e^{j\theta_u}s_u(k) + b_u(k), \quad (15)$$

where the index $v$ depends on the permutation factor and where $b_u(k)$ is an additive Gaussian noise. We propose to estimate the unknown phase factor,
\[ \hat{\theta}_u(k) = \frac{1}{q} \left( \mathbb{E}[s^q] \sum_{k=0}^{N-4} (y_u^S(k))^q \right). \]  

(16)

The order \( q \) and \( \mathbb{E}[s^q] \) depend on constellation assumption. For normalized \( M \)-PAM, the constellation is \( \pi \)-rotationally symmetric so \( q = 2 \) and \( \mathbb{E}[s^2] = 1 \) since the energy of the transmitted symbols are normalized. For \( M \)-PSK, \( q = M \) and \( \mathbb{E}[s^q] = 1 \). For rectangular \( M \)-QAM, \( q = 4 \) and \( \mathbb{E}[s^4] \) depends on the number of states \( M \). To illustrate this second stage, let us consider a MIMO-SM system using \( nt = 2 \), \( nr = 2 \) antennas and a QPSK modulation. Figure 2a and Figure 2b show the constellation of the two received signals, \( y_1(k) \) and \( y_2(k) \). Figure 2c and Figure 2d display the constellation of the separated components after ICA. Compared to QPSK constellation, the constellations of the separated components are phase-rotated. In Figure 2e and Figure 2f, we can remark that the phase correction algorithm adapts the constellation of the separated components, \( e^{-j\hat{\theta}_1} y_1^S(k) \) and \( e^{-j\hat{\theta}_2} y_2^S(k) \), to the QPSK one.

Because of the constellation properties, the second stage is able to recover the phase up to an unknown factor \( e^{-2j\pi\rho_u/q} \), where \( \rho_u \) is an integer. Taking into account this indetermination, the channel matrix can be expressed as:

\[ \hat{H} = \hat{H}\hat{D}_q P, \]  

(17)

with:

\[ \hat{D} = \begin{pmatrix} e^{j\hat{\theta}_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{j\hat{\theta}_{nr}} \end{pmatrix}, \]  

(18)

\[ D_q = \begin{pmatrix} e^{2j\pi\rho_1/q} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{2j\pi\rho_{nr}/q} \end{pmatrix}. \]  

(19)
Figure 2. Constellation of the received samples separated component for a MIMO communication using \( nt = nr = 2 \) antennas and a QPSK modulation (\( SNR = 20 \) dB, \( N = 512 \) samples)

Theorem 2: Under the assumption that the transmitted symbols belong to constellation \( \Omega_u \), the matrix \( D_q P \), where \( D_q \) is defined in Equation (19) and \( P \) is a permutation matrix, does not modify the Likelihood Function, i.e.:

\[
\log(\Delta[Y/\Omega_u, H]) = \log(\Delta[Y/\Omega_u, \hat{H}\hat{D}]).
\]

Proof: First from Theorem 1, we obtain

\[
\log(\Delta[Y/\Omega_u, H]) = \log(\Delta[Y/\Omega_u, \hat{H}\hat{D} D_q])
\]

Then, let us focus on the terms:

\[
\sum_{S(k) \in \Omega''_u} \exp\left[ -\frac{1}{\sigma^2} ||Y(k) - \hat{H}\hat{D} D_q S(k)||_F^2 \right].
\]
Under the assumption that the symbols belong to $\Omega_u$, which is $2\pi/Q$-rotationally symmetric, $S(k)$ and $D_qS(k)$ belong to the same set $\Omega_u^{nt}$. Setting $D_q = I_{nt}$ permutes the elements of $\Omega_u^{nt}$ and does not change the sum result.

Finally, the Likelihood Function can be approximated by $\log(\Delta[Y/\Omega_u, \hat{H}\hat{D}])$. Algorithm 1 sums up the proposed method.

```
• $\log\Delta_{\text{max}} \leftarrow -\infty$
• compute $\hat{H}$ with JADE algorithm
  for $\Omega_u \in \Theta$
    • compute the components $Y^\eta(k) = [y^\eta_1(k) \ldots y^\eta_{nu}(k)]^T$ with Equation (13)
    • estimate the phase $\theta^\eta(k)$ for each separated components with Equation (15)
    • construct the matrix $\hat{D}$ with Equation (17)
    • compute the log-LF $\log(\Delta[Y/\Omega_u, \hat{H}\hat{D}])$ with Equation (8)
      if $\log(\Delta[H/\Omega_u, \hat{H}\hat{D}]) \geq \log\Delta_{\text{max}}$
        • $\log\Delta_{\text{max}} \leftarrow \log(\Delta[Y/\Omega_u, \hat{H}\hat{D}])$
        • $\hat{\Omega} \leftarrow \Omega_u$
      endif
  endfor
```

5. Simulation Results

In this section, we present the performances of the two proposed approaches for the recognition of the 4 modulation formats: BPSK, QPSK, 16PSK and 16QAM. 500 Monte Carlo trials were performed for each modulation format to approximate the probability of detection. Moreover, the conditions for each trial were: i) a MIMO system composed of $nt = 2$ and $nr = 4$ antennas; ii) a Rayleigh distributed channel, which means that each element of $H$ follows a complex Gaussian circular law with zero mean and unit variance; iii) 512 i.i.d. transmitted symbols on each antenna; iv) a spatially white Gaussian noise of variance $\sigma^2$; v) a signal to noise ratio (SNR) equals to $10\log(n/t/\sigma^2)$, and vi) a perfect knowledge of the number of transmitters and of the noise variance at the receiver side.

In Subsections 5.1, we present the performances of our proposed algorithms with and without channel knowledge at the receiver side.
5.1 Performances with channel knowledge

Figure 3 presents the performances of the first classifier with channel knowledge at the receiver side. This classifier is optimal in the Bayesian sense and, so provides an upper bound of classification.

Figure 3a displays the probability of detection of each modulation format when the transmitted symbols belong to a BPSK modulation, i.e., $P[\hat{\Omega}/\Omega = \text{BPSK}]$. We remark that the BPSK modulation is perfectly detected for a $\text{SNR}$ equals to $-10$ dB.

Figure 3b exposes the performances when the transmitter uses a QPSK modulation. The modulation is correctly detected for a $\text{SNR}$ equals to $0$ dB. At low $\text{SNR}$, we remark ambiguities between QPSK, 16PSK and 16QAM modulations. At $\text{SNR} = -5$ dB, the main false detection occurs when a 16PSK
constellation is recognized instead of the QPSK one. In Figure 3c, the same simulation is performed with 16PSK modulation. For $SNR < 0$, we observe confusions between QPSK, 16QAM and 16PSK modulations. A perfect detection is achieved for $SNR = 0$ dB. Finally, Figure 3d exposes the probability of detection when the transmitted symbols belong to a 16QAM modulation. We note that the modulation is correctly detected at a $SNR$ of 2 dB.

5.2 Performances without channel knowledge

Figure 4 presents the performances obtained with the Algorithm 1. This sub-optimal method does not require the channel knowledge at the receiver side.

![Graphs comparing different modulation formats](image)

Figure 4. Performances with unknown channel matrix; MIMO communication using $nt = 2$ and $nr = 4$ antennas; 512 received samples per antenna

Figure 4a displays the probability of recognition of each modulation format when the transmitted symbols belong to a BPSK modulation. While a perfect detection is obtained at a $SNR = -10$ dB with the optimal classifier, the
probability of BPSK detection is close to 1 at $SNR = 1 \text{ dB}$. **Figure 4a** shows that most of the confusions occur when a QPSK or a 16QAM format is detected instead of a BPSK one. **Figure 4b** shows the performances for QPSK modulations. This modulation is correctly detected at a $SNR$ equals to 5 dB. There is a difference of 5 dB compared to the optimal performances. The most important confusions occur when the algorithm selects 16QAM constellation instead of the QPSK one. **Figure 4c** presents the probability of detection when the symbols belong to a 16PSK constellation. At low $SNR$, we remark confusions between QPSK, 16PSK and 16QAM modulations. Our proposed classifier recognizes the QPSK modulation from $-5 \text{ dB}$. The probability of correct detection is close to 1 at a $SNR$ equals to 5 dB (0 dB for the optimal classifier). Finally, **Figure 4d** displays the performances for 16QAM modulation. We remark that the performances are close to the optimal ones for high $SNR$: a perfect detection is achieved at $SNR = 2 \text{ dB}$. However at low $SNR$, the classifier fails to discriminate QPSK and 16QAM constellation.

### 6. Conclusion

This paper investigated the problem of the blind recognition of the modulation for MIMO systems using Spatial Multiplexing. Two Likelihood-based classifiers were proposed. The first one, called ALRT, is optimal in the Bayesian sense but requires the knowledge of the channel matrix. The second classifier, called HLRT, approximates the ALRT by replacing the channel matrix with its estimate. The channel is estimated in two steps by using an Independent Component Analysis and a phase Correction algorithm respectively. The performances of the two methods were evaluated for MIMO communications using 2 transmitter and 4 receiver antennas. The simulations showed that the two classifiers perform well and, for example, perfectly recognize BPSK, QPSK, 16PSK and 16QAM modulations at a $SNR$ equals to 5 dB.

Future works will be devoted to the blind recognition of the modulation for MIMO systems using Space-Time Coding.

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### References


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