Blind Recognition of Linear Space Time Block Codes

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ABSTRACT

The blind recognition of communication parameters is a key research issue for commercial and military communication systems. In this paper, we investigate the problem of the blind recognition of Linear Space-Time Block Codes (STBC). To characterize the space time coding, we propose to compute a time-lag correlation of the received samples. Provided the number of transmitters, the noise variance and the symbol timing are well estimated, we show that the theoretical values of the correlation norm only depend on the STBC and are affected by neither the channel nor the symbol modulation. The automatic recognition of the STBC is realized by selecting the STBC which minimizes the distance between the theoretical values and the experimental ones. Simulations show that our method performs well even for low signal to noise ratio (0dB).

Index Terms— Signal Detection, MIMO Systems, Electronic Warfare, Space time coding

1. INTRODUCTION

The blind recognition of the communication parameters is an important research topic with application in both commercial and military communication. The blind recognition permits the receiver to characterize the intercepted communication when the protocol is unknown. In the last decade, a lot of work has been done to blindly detect the number of transmitter antennas [1], the modulation [2][3], and to blindly equalize the received signal [4], etc. As a result of emerging wireless technologies, new problems of recognition have recently raised. One promising technology is based on multiple antennas like Multiple-Input Multiple-Output (MIMO) communications. These communications use space-time coding at the transmitter side to improve the robustness of the transmission. A large amount of Space Time Codes (STC) is exposed in literature, an overview is presented in reference [5]. To our knowledge, no method exists in literature for the blind recognition of STC. In this paper, we propose a new method to recognize linear Space Time Block Codes (STBC) which are the major category of STC.

This article is organized as follows. In Section II, we present the signal model of the communication. The section III presents a preprocessing step based on whitening which permits to cancel the influence of the unknown propagation channel. In section IV and V, we expose respectively some discriminating features of the STBC based on a time-lag correlation and the associated classifier. Finally, the performances of our proposed method are shown in section VI.

2. SIGNAL MODEL

2.1. STBC signal model

Let us consider a vector $S_v = [s_1, \cdots, s_{n_s}]$ composed of n_s i.i.d complex symbols which belong to a (≥ 4)PSK or (≥ 4)QAM modulation. For a linear STBC, the vector S_v is transmitted during l slots according to a transmission matrix, $\mathbf{C}(S_v)$ of size $n_t \times l$ which is defined by:

$$\mathbf{C}(S_v) = [C_0(S_v) \cdots C_{l-1}(S_v)] \tag{1}$$

where $C_j(S_v)$ is the $(j+1)^{th}$ transmitted column vector of size n_t which can be expressed with respect to the vector S_v as:

$$C_u(S_v) = \begin{bmatrix} \mathbf{A}_u^{(\Re)} & i\mathbf{A}_u^{(\Im)} \end{bmatrix} \times \tilde{S}_v \tag{2}$$

where $\tilde{S}_v$ is a column vector of size $2n_s$ which corresponds to the concatenation of the real and imaginary part of the vector S_v, i.e. $\tilde{S}_v = [\Re(S_v)\ \Im(S_v)]^T$. The matrices $\mathbf{A}_j^{(\Re)}$ and $\mathbf{A}_j^{(\Im)}$ only depend on the STBC and are composed of $n_t \times n_s$ real elements. For example, for an Alamouti coding [6] these matrices are equal to:

$$\begin{bmatrix} \mathbf{A}_0^{(\Re)} & i\mathbf{A}_0^{(\Im)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & i & 0 \\ 0 & 1 & 0 & i \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{A}_1^{(\Re)} & i\mathbf{A}_1^{(\Im)} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & i \\ 1 & 0 & -i & 0 \end{bmatrix}$$

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2.2. Signal model of the intercepted communication

Let us consider a communication using linear STBC of size $n_t \times l$ at the transmitter side. Let us denote by the vector $X(k)$ ($0 \leq k < N$), the $(k+1)^{th}$ column which is transmitted over the n_t antennas. The column $X(k)$ can be expressed with respect to $C_u(S_v)$ as:

$$X(k) = C_{k\text{mod }l}(S_{k\text{div }l}) \tag{3}$$

where $k$div l and $k$mod l correspond respectively to the quotient and the reminder of the division $\frac{k}{l}$. Let us assume a blind receiver composed of n_r antennas ($n_r \geq n_t$). Under the assumptions that the timing symbol is well estimated at the receiver side and that the channel, denoted by a $n_r \times n_t$ matrix $\mathbf{H}$, is time-invariant during the interception and frequency-flat, the $(k+1)^{th}$ intercepted samples can be expressed as:

$$\begin{aligned} \begin{bmatrix} \Re(R(k)) \\ \Im(R(k)) \end{bmatrix} &= \begin{bmatrix} \Re(\mathbf{H}) & -\Im(\mathbf{H}) \\ \Im(\mathbf{H}) & \Re(\mathbf{H}) \end{bmatrix} \begin{bmatrix} \Re(X(k)) \\ \Im(X(k)) \end{bmatrix} \\ &+ \begin{bmatrix} \Re(B(k)) \\ \Im(B(k)) \end{bmatrix} \end{aligned} \tag{4}$$

The column vector $B(k)$ of size n_r represents the additive noise which is considered both spatially and temporally white with variance σ^2. $R(k)$ and $X(k)$ are respectively a column vector of size n_r which corresponds to the received samples and a column vector of size n_t which corresponds to the transmitted signals. The equation (4) can be expressed in the following compact form:

$$\tilde{R}(k) = \breve{\mathbf{H}}\tilde{X}(k) + \tilde{B}(k) \tag{5}$$

where $\breve{\mathbf{H}}$ is a matrix of size $2n_r \times 2n_t$ and where $\tilde{.}$ denotes the concatenation of real and imaginary parts. The correlation matrix $E[\tilde{X}(k)\tilde{X}^T(k)]$ is assumed to be a diagonal matrix with equal diagonal elements, i.e. $E[\tilde{X}(k)\tilde{X}^T(k)] = \alpha^2\mathbf{I}_{2n_t}$ where $\mathbf{I}_{2n_t}$ is an identity matrix of size $2n_t \times 2n_t$ and α^2 corresponds to the transmitted energy on each antenna.

In the following sections, we expose a method for the blind recognition of an STBC from the samples $R(k)$ ($0 \leq k < N$) provided the noise variance σ^2 and the number of transmitters n_t are known or well estimated.

3. PREPROCESSING BASED ON WHITENING

The first part of our method consists in whitening the signal part of the observation to cancel the influence of the channel matrix. The whitening is achieved by applying to the signal $\tilde{R}(k)$ a whitening matrix $\mathbf{W}$ of size $2n_t \times 2n_r$ verifying:

$$E[\mathbf{W}\breve{\mathbf{H}}\tilde{X}(k)\tilde{X}^T(k)\breve{\mathbf{H}}^T\mathbf{W}^T] = \alpha^2\mathbf{W}\breve{\mathbf{H}}\breve{\mathbf{H}}^T\mathbf{W}^T = \mathbf{I}_{2n_t} \tag{6}$$

where α^2 corresponds to the transmitted power. A whitening matrix $\mathbf{W}$ can be extracted from the $2n_r \times 2n_r$ covariance matrix $E[\tilde{R}(k)\tilde{R}^T(k)] - \frac{\sigma^2}{2}\mathbf{I}_{2n_r}$. Let's denote by Λ and $\mathbf{U}$ respectively the $2n_t \times 2n_t$ diagonal matrix which contains the $2n_t$ largest eigenvalues of the covariance matrix and the $2n_r \times 2n_t$ unitary matrix which contains the associated eigenvectors, a whitening matrix is given by:

$$\mathbf{W} = \Lambda^{-1/2}\mathbf{U}^T \tag{7}$$

The whitened process $Y(k) = \mathbf{W}\tilde{R}(k)$ obeys a linear model which can be expressed with respect to the signal $\tilde{X}(k)$ as [7]:

$$Y(k) = \frac{1}{\alpha}\mathbf{MKP}\tilde{X}(k) + \mathbf{W}\tilde{B}(k) \tag{8}$$

where $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{P}$ are unknown matrices of size $2n_t \times 2n_t$ which are respectively an unitary matrix, a diagonal sign matrix and a permutation matrix. In the next section, we propose to characterize the STBC from a time-lag correlation of the whitened process $Y(k)$.

4. ANALYSIS OF THE TIME LAG CORRELATION

As space time coding introduces space-time redundancy, we propose to detect the STBC of the intercepted communication with the following time-lag correlation:

$$\mathbf{R}_Y(\tau) = \lim_{N\to\infty}\frac{1}{N}\sum_{k=0}^{N-1}E[Y(k)Y^T(k+\tau)] \tag{9}$$

Using the equation (8) and the fact that the noise is temporally white, for a non null time lag ($\tau \neq 0$) this correlation is equal to:

$$\mathbf{R}_Y(\tau) \quad = \quad \frac{1}{\alpha^2}\mathbf{MKPR}_{\tilde{X}}(\tau)\mathbf{P}^T\mathbf{K}^T\mathbf{M}^T \tag{10}$$

where $\mathbf{R}_{\tilde{X}}(\tau)$ corresponds to the time lag correlation of the transmitted signals defined by:

$$\mathbf{R}_{\tilde{X}}(\tau) = \lim_{N\to\infty}\frac{1}{N}\sum_{k=0}^{N-1}E[\tilde{X}(k)\tilde{X}^T(k+\tau)] \tag{11}$$

The link between the transmitted signals and the transmitted blocks is given by equation (1). As the modulated symbols to be encoded in two different vectors S_v are independent and as the number of transmitted vector is asymptotically equal to $\frac{N}{l}$, the equation (11) can be expressed as:

$$\mathbf{R}_{\tilde{X}}(\tau) = \lim_{N\to\infty}\frac{1}{N}\sum_{v=0}^{\frac{N}{l}-1}\sum_{u=0}^{l-\tau-1}E[\tilde{C}_u(S_v)\tilde{C}^T_{u+\tau}(S_v)] \tag{12}$$

Using equation (2) and the fact that the symbols s are identically distributed, we can remark that the statistical properties of $E[\tilde{C}_u(S_v)\tilde{C}^T_u(S_{v+\tau})]$ do not depend on the block number and so v can be assigned to a constant value. Setting $v = 1$ in equation (12) leads to:

$$\mathbf{R}_{\tilde{X}}(\tau) = \frac{1}{l}\sum_{u=0}^{l-\tau-1} E[\tilde{C}_u(S_1)\tilde{C}_{u+\tau}^T(S_1)] \quad (13)$$

This correlation can be expressed with respect to the construction matrices of the code using equation (2) in (13). Using the fact that for (≥ 4)PSK or (≥ 4)QAM modulations $E[\tilde{S}_1\tilde{S}_1^T] = \frac{E[|s|^2]}{2}\mathbf{I}_{2n_s}$, the correlation $\mathbf{R}_{\tilde{X}}(\tau)$ is equal to:

$$\mathbf{R}_{\tilde{X}}(\tau) = \frac{E[|s|^2]}{2l}\mathbf{R}_c(\tau) \quad (14)$$

where $\mathbf{R}_c(\tau)$ is a $2n_t \times 2n_t$ matrix which is defined by:

$$\mathbf{R}_c(\tau) = \sum_{u=0}^{l-\tau-1}\begin{bmatrix} \mathbf{A}_u^{(\Re)}\mathbf{A}_{u+\tau}^{(\Re),T} & \mathbf{0}_{n_s} \\ \mathbf{0}_{n_s} & \mathbf{A}_u^{(\Im)}\mathbf{A}_{u+\tau}^{(\Im),T} \end{bmatrix} \quad (15)$$

and where $\mathbf{0}_{n_s}$ is a null matrix of size $n_s \times n_s$. According to equation (10) and (14), the time lag correlation of the whitened process is given by:

$$\mathbf{R}_Y(\tau) = \frac{E[|s|^2]}{2l\alpha^2}\mathbf{MKPR}_c(\tau)\mathbf{P}^T\mathbf{K}^T\mathbf{M}^T \quad (16)$$

It is difficult to recognize the matrix $\mathbf{R}_c(\tau)$ from the matrix $\mathbf{R}_Y(\tau)$ since the latter matrix depends on unknown unitary matrices. To overcome this problem, we propose to compute the Frobenius norm of the matrix $\mathbf{R}_Y(\tau)$. Using the fact that $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{P}$ are unitary matrices, the Frobenius norm of the time-lag correlation is equal to:

$$||\mathbf{R}_Y(\tau)||_F^2 = \frac{E[|s|^2]^2}{4l^2\alpha^4}||\mathbf{R}_c(\tau)||_F^2 \quad (17)$$

where $||.||_F^2$ denotes the Frobenius norm. The transmitted power can be expressed as: $\alpha^2 = (||E[\tilde{X}(k)\tilde{X}^T(k)]||_F^2/2n_t)^{\frac{1}{2}}$. According to equations (14) and (11), α^4 is given by:

$$\alpha^4 = \frac{E[|s|^2]^2}{8l^2n_t}||\mathbf{R}_c(0)||_F^2 \quad (18)$$

Finally, using equation (18) in (17) leads to the following expression of the Frobenius norm:

$$||\mathbf{R}_Y(\tau)||_F^2 = 2n_t\frac{||\mathbf{R}_c(\tau)||_F^2}{||\mathbf{R}_c(0)||_F^2} \quad (19)$$

The Frobenius norm of the matrix $||\mathbf{R}_Y(\tau)||_F^2$ only depends on the space time coding. Neither the channel matrix nor the modulation affect it. To illustrate this property, the figure 1 shows the experimental values of the Frobenius norm for an intercepted communication using an orthogonal STBC of rate $\frac{1}{2}$ with 3 antennas [8]. The theoretical values defined by equation (19) are plotted on the same figure. It is worth noting that the experimental values are well approximated by the theoretical ones. In the next section, we propose a classifier which permits to automatically recognize the STBC from the experimental values of $||\mathbf{R}_Y(\tau)||_F^2$.

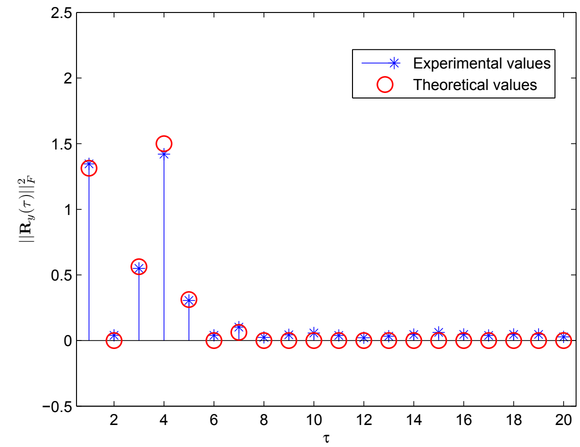

Fig. 1. Theoretical and experimental values of $||\mathbf{R}_Y(\tau)||_F^2$ for a communication using Orthogonal STBC of rate $\frac{1}{2}$. The transmitter and the receiver are respectively composed of 3 and 4 antennas. 1024 QPSK symbols are transmitted on each antenna and the Signal to Noise Ratio is equal to 10dB.

5. AUTOMATIC CLASSIFICATION

In practise the Frobenius norm of the time-lag correlation is estimated by the circular statistic $||\widehat{\mathbf{R}}_Y(\tau)||_F^2$ defined by:

$$||\widehat{\mathbf{R}}_Y(\tau)||_F^2 = \frac{1}{N^2}\left|\left|\sum_{k=0}^{N-1} Y(k)Y((k+\tau)\mathrm{mod}\ N)\right|\right|_F^2 \quad (20)$$

where N is the number of intercepted columns $Y(k)$. To automatically recognize the STBC of the whitened process in a set $\mathcal{C}$ of STBC candidates, we propose to compute a distance between the experimental values $||\widehat{\mathbf{R}}_Y(\tau)||_F^2$ and the theoretical ones for each possible code $c \in \mathcal{C}$. The proposed distance, d_c, is given by:

$$\mathrm{d}_c = \sum_{\tau=1}^{l_{max}}\left(||\widehat{\mathbf{R}}_Y(\tau)||_F^2 - \max\left(2n_t\frac{||\mathbf{R}_c(\tau)||_F^2}{||\mathbf{R}_c(0)||_F^2}, \epsilon\right)\right)^2 \quad (21)$$

where l_{max} is the maximum code length l of the set $\mathcal{C}$ and where ϵ is a threshold value below which the theoretical values are not significative over the estimation errors of $||\widehat{\mathbf{R}}_Y(\tau)||_F^2$. This threshold can be simply set to the average value of $||\widehat{\mathbf{R}}_Y(\tau)||_F^2$ for which the theoretical values are null, i.e.

$$\epsilon = \frac{1}{N - l_{max} - 1}\sum_{l_{max}+1}^{n}||\widehat{\mathbf{R}}_Y(\tau)||_F^2 \quad (22)$$

Finally, the recognized STBC, $\widehat{c}$, is the one which minimizes the distance d_c, i.e.:

$$\widehat{c} = \arg\min_{c\in\mathcal{C}} \mathrm{d}_c \quad (23)$$

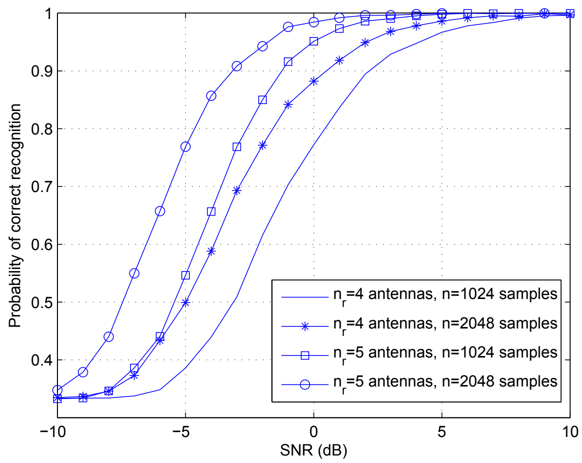

Fig. 2. Probability of correct recognition with respect to the SNR for 3 types of STBC using 3 antennas: Spatial Multiplexing, OSTBC of rate $\frac{3}{4}$ and OSTBC3 of rate $\frac{1}{2}$.

In the next section, we present some simulations to highlight the behavior of our proposed algorithm.

6. SIMULATION RESULTS

The probability of correct recognition of our proposed algorithm is evaluated for the recognition of 3 types of STBC using 3 antennas: Spatial Multiplexing (SM), Orthogonal STBC of rate $\frac{1}{2}$ [8] and Orthogonal STBC of rate $\frac{3}{4}$ [5]. 1000 Monte carlo simulations were performed for each type of code and the probability of correct recognition is approached by averaging the total number of correct recognitions over the total number of trials. Moreover, the conditions for each Monte Carlo trial were: i) a Rayleigh distributed channel $\mathbf{H}$, ii) a QPSK modulation for the transmitted symbols, iii) a complex gaussian circular noise, $B(k)$ and iv) a perfect knowledge of the number of transmitters, n_t, of the noise variance, σ^2, and of the symbol timing at the receiver side.

The figure 2 shows the probability of correct recognition with respect to the Signal to Noise Ratio (SNR). The probability of correct recognition for $n = 1024$ received samples and $n_r = 4$ receivers is close to 1 at a SNR equals to 10dB. We can remark that increasing the number of receivers and/or the number of received samples, significantly improves the STBC recognition. For a receiver composed of $n_r = 5$ antennas intercepting 2048 samples, the probability of correct recognition is close to 1 at 0dB. In fact, a larger number of receiver antennas increases the noise removal of the whitened process $Y(k)$. Furthermore, a larger number of received samples leads to a better estimate of the statistic $||\widehat{\mathbf{R}}_Y(\tau)||_F^2$.

7. CONCLUSION

This paper described a method for the blind recognition of Linear Space-Time Block Codes. First, the intercepted samples are whitened, then the characterization of the STBC is obtained from a time-lag correlation of the whitened process. We showed that the correlation norm only depends on the construction matrices of the STBC. Finally, the automatic recognition of the STBC is realized by comparing the theoretical and experimental values of this correlation norm. The performances of our method were evaluated for the recognition of 3 STBC using 3 antennas: Spatial Multiplexing and 2 Orthogonal STBC with rate of $\frac{3}{4}$ and $\frac{1}{2}$ respectively. Experimental results showed good performances even at low SNR (0dB). These performances were enhanced by elevating the number of receiver antennas and/or that of received samples.

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